Games where you can play optimally with finite memory

Mickael Randour

F.R.S.-FNRS & UMONS – Université de Mons, Belgium

April 16, 2021

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A sequel to the critically acclaimed blockbuster by Gimbert & Zielonka

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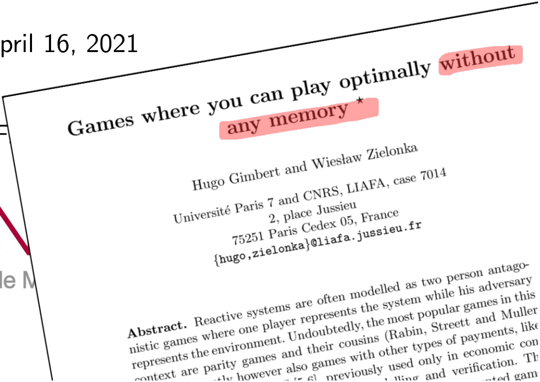

Games where you can play optimally without any memory *

Hugo Gimbert and Wiesław Zielonka

Université Paris 7 and CNRS, LIAFA, case 7014
2, place Jussieu
75251 Paris Cedex 05, France
{hugo,zielonka}@liafa.jussieu.fr

Abstract. Reactive systems are often modelled as two person antago-
nistic games where one player represents the system while his adversary
represents the environment. Undoubtedly, the most popular games in this
context are parity games and their cousins (Rabin, Streett and Muller
... however also games with other types of payments, like
... previously used only in economic con
... and verification. Th

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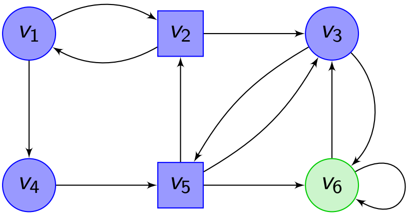
We consider *finite* arenas with vertex *colors* in C. Two players: circle ($\mathcal{P}_1$) and square ($\mathcal{P}_2$). Strategies $C^* \times V_i \to V$.



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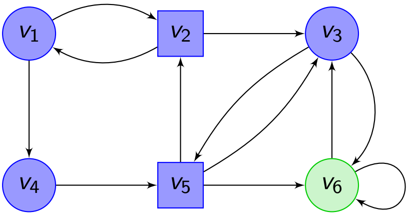

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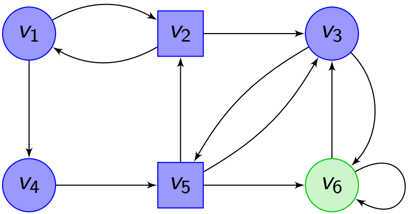

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Memoryless strategies ($V_i \rightarrow V$) always suffice for reachability (for both players).

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Virtually always for **simple** winning conditions!

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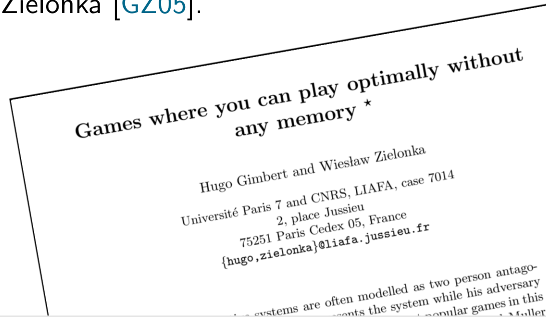
Yes, thanks to Gimbert and Zielonka [GZ05].



Gimbert and Zielonka's characterization

Memoryless strategies suffice for a *preference relation* $\sqsubseteq$ (and the induced winning conditions) **if and only if**

1. it is **monotone**,

2. it is **selective**.

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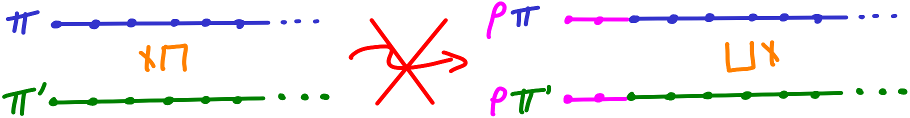

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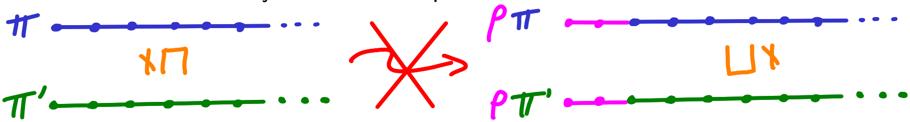

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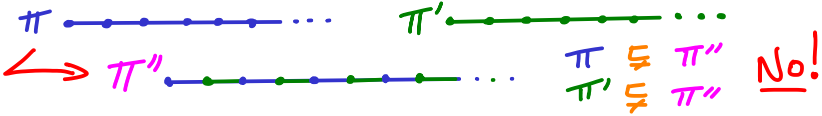

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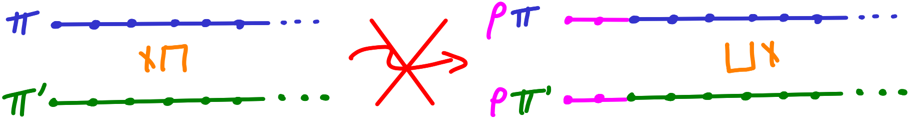

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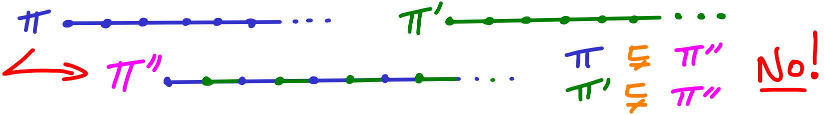

Example: reachability.

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then both players have optimal memoryless strategies **in all two-player arenas**.

Extremely useful in practice!

Going further: finite memory

Memoryless strategies do not always suffice!

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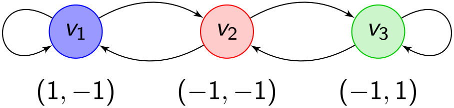

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- Büchi for v_1 *and* v_3 → **finite** (1 bit) memory.

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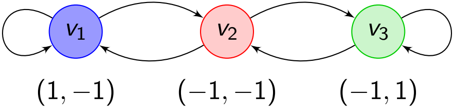

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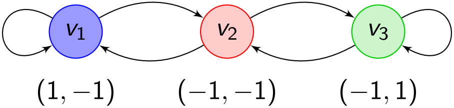

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We need a GZ equivalent for finite memory!

$\rightsquigarrow$ For *combinations*, see [LPR18].

A partial counter-example (lifting corollary)

Let $C \subseteq \mathbb{Z}$ and the winning condition for $\mathcal{P}_1$ be

$$\overline{TP}(\pi) = \infty \quad \vee \quad \exists^{\infty} i \in \mathbb{N}, \sum_{i=0}^{n} c_i = 0$$

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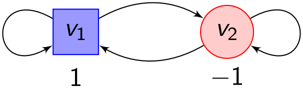

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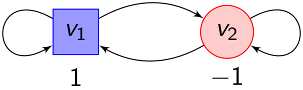

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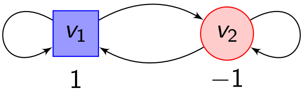

⚡ No exact equivalent to GZ ⚡

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A new frontier

Arena-independent finite memory

The *memory skeleton* $\mathcal{M}$ only depends on the preference relation, not on the (size of the) graph.

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We obtain a natural GZ-equivalent for (AI)FM determinacy, including the lifting corollary (1-p. to 2-p.)!

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$\hookrightarrow$ **Follow-up:** extension to **stochastic games** with Bouyer, Oualhadj and Vandenhove [BORV21].

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/!\ Much more involved technically.

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Thank you! Any question?

References I

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