Average-Energy Games

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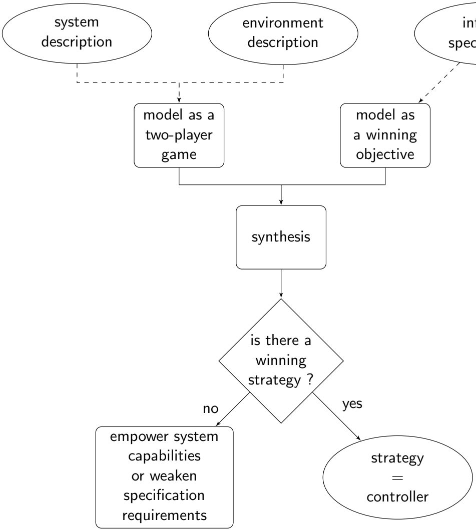
09.06.2015 - *Séminaire annuel du LSV* - Dourdan

General context: strategy synthesis in quantitative games



1. How complex is it to decide if a winning strategy exists?
2. How complex such a strategy needs to be? **Simpler is better**.
3. Can we synthesize one efficiently?

$\Rightarrow$ Depends on the *winning objective*.

The talk in one slide

- **"New" quantitative objective**
  - Total-payoff (TP) "refines" mean-payoff (MP) (MP value $= 0$)
  - **Average-energy (AE)** "refines" TP

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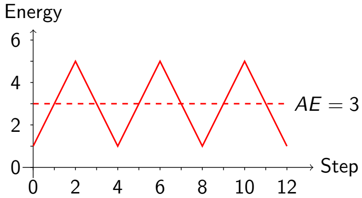
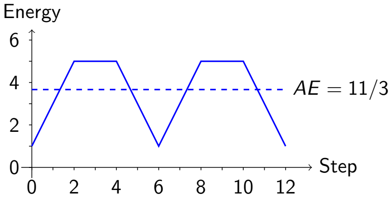
  - ▷ Total-payoff (TP) "refines" mean-payoff (MP) (MP value = 0)
  - ▷ **Average-energy (AE)** "refines" TP
  - ↪ characterizes the **average energy level** along an infinite play





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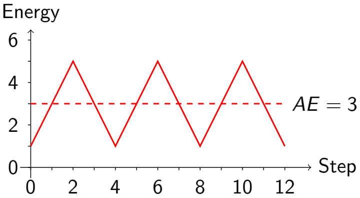
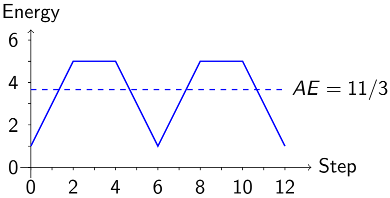
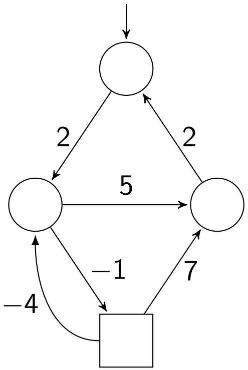




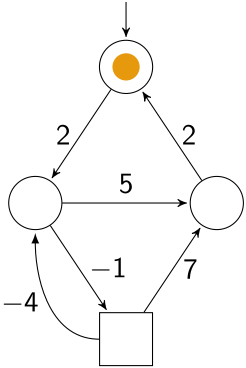
- Conjunction with **energy constraints**: lower and/or upper bounds on the energy level (e.g., fuel tank)

Two-player turn-based games on graphs

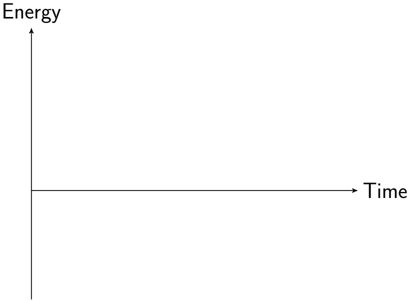


- $G = (S_1, S_2, T, w)$
- $S = S_1 \cup S_2$, $S_1 \cap S_2 = \emptyset$, $T \subseteq S \times S$, $w \colon T \to \mathbb{Z}$
- $\mathcal{P}_1$ states $= \bigcirc$
- $\mathcal{P}_2$ states $= \square$
- Plays have values
  - ▷ $f\colon \mathit{Plays}(G) \to \mathbb{R} \cup \{-\infty, \infty\}$
- Players follow **pure** strategies
  - ▷ $\sigma_i \colon \mathit{Prefs}_i(G) \to S$

Energy, total-payoff, mean-payoff



- $EL(\pi(n)) = \sum_{i=0}^{n-1} w(s_i, s_{i+1})$
- $\overline{TP}(\pi) = \limsup_{n\to\infty} EL(\pi(n))$
- $\overline{MP}(\pi) = \limsup_{n\to\infty} \frac{1}{n} EL(\pi(n))$



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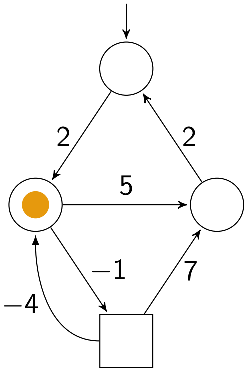

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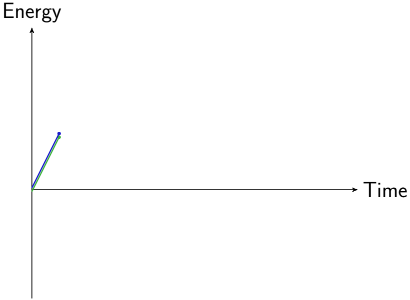

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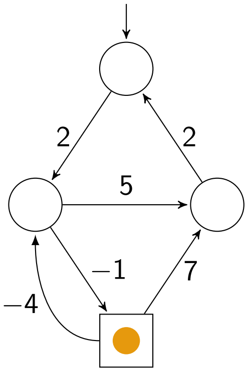

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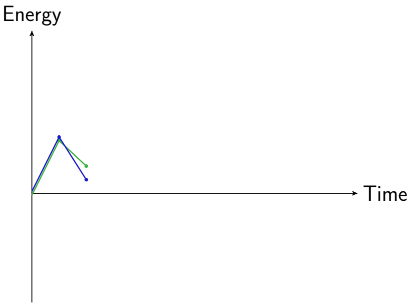

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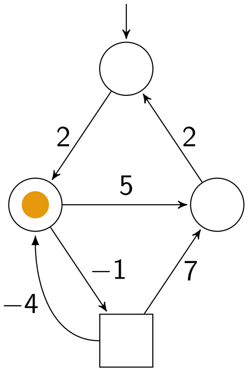

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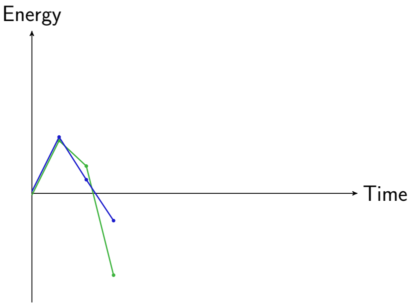

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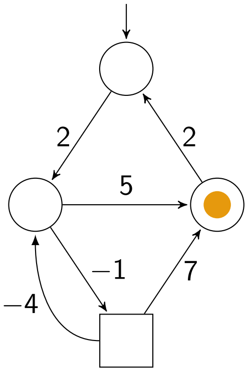

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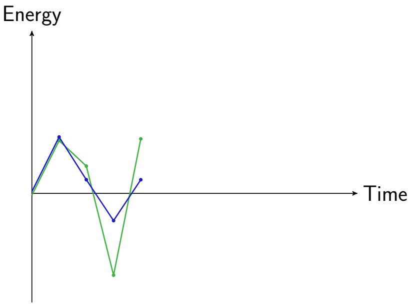

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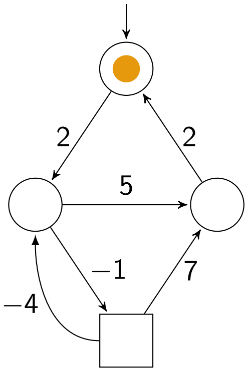

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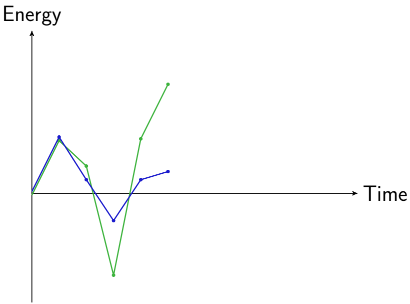

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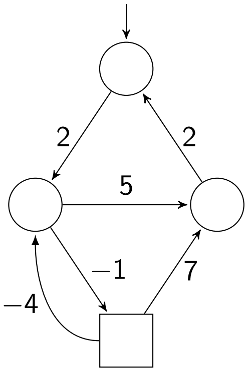

Then, $(2, 5, 2)^\omega$

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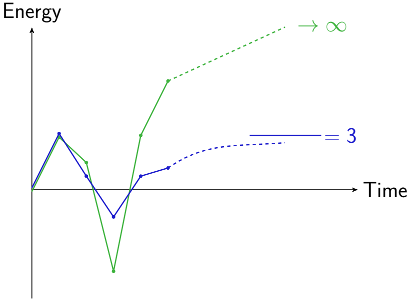


Decision problems

- **TP (MP) threshold problem**
  - ▷ Given $t \in \mathbb{Q}$ and $s_{\mathsf{init}} \in S$, $\exists ?\, \sigma_1 \in \Sigma_1$ s.t. $\forall \sigma_2 \in \Sigma_2$,
    $\overline{TP}(Outcome(s_{\mathsf{init}}, \sigma_1, \sigma_2)) \leq t$
  - ↪ we take the **minimizer** point of view
- **Lower-bounded energy problem**
  - ▷ Given $c_{\mathsf{init}} \in \mathbb{N}$ and $s_{\mathsf{init}} \in S$, $\exists ?\, \sigma_1 \in \Sigma_1$ s.t. $\forall \sigma_2 \in \Sigma_2$,
    $\forall n \geq 0,\ c_{\mathsf{init}} + EL(Outcome(s_{\mathsf{init}}, \sigma_1, \sigma_2)(n)) \geq 0$
  - ↪ **fixed** initial credit
- **Lower- and upper-bounded energy problem**
  - ▷ Given $c_{\mathsf{init}} \in \mathbb{N}$, $U \in \mathbb{N}$ and $s_{\mathsf{init}} \in S$, $\exists ?\, \sigma_1 \in \Sigma_1$ s.t. $\forall \sigma_2 \in \Sigma_2$,
    $\forall n \geq 0,\ c_{\mathsf{init}} + EL(Outcome(s_{\mathsf{init}}, \sigma_1, \sigma_2)(n)) \in [0, U]$

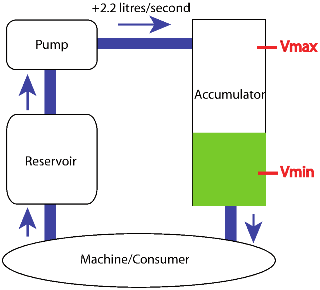
Known results

Objective	**1-player**	**2-player**	**memory**
MP	P [Kar78]	NP $\cap$ coNP [ZP96]	memoryless [EM79]
TP	P [FV97]	NP $\cap$ coNP [GS09]	memoryless [GZ04]
EG_L	P [BFL+08]	NP $\cap$ coNP [CdAHS03, BFL+08]	memoryless [CdAHS03]
EG_{LU}	PSPACE-c. [FJ13]	EXPTIME-c. [BFL+08]	pseudo-polynomial

- For all objectives but EG_{LU}, *memoryless* strategies suffice for both players.

Average-energy: motivating example

HYDAC oil pump industrial case study [CJL+09] (Quasimodo research project).



Goals:

1. Keep the oil level in the safe zone.
   $\hookrightarrow$ EG_{LU}
2. Minimize the average oil level.
   $\hookrightarrow$ AE

$\Rightarrow$ **Conjunction:** AE_{LU}

Related work

- *AE* appeared in [TV87] as an alternative *total reward* definition.
  - ↪ Not studied until recently.
- Chatterjee and Prabhu use a variant in [CP13].
  - ↪ *Average debit-sum level* objective.
  - ↪ Pseudo-polynomial algorithm.
  - ↪ Complexity and memory requirements are open.
- *AE* studied independently by Boros et al. in [BEGM15].
  - ↪ Stochastic context.
  - ↪ Similar results but different approach.
- Nothing is known for AE_{LU}.

Average-energy: definition

Recall

- $EL(\pi(n)) = \sum_{i=0}^{n-1} w(s_i, s_{i+1})$
- $\overline{TP}(\pi) = \limsup_{n\to\infty} EL(\pi(n))$
- $\overline{MP}(\pi) = \limsup_{n\to\infty} \frac{1}{n} EL(\pi(n))$

+ infimum variants
$\underline{TP}$, $\underline{MP}$, $\underline{AE}$

Average-energy (AE)

Describes the **average energy level** along a play:

$$\overline{AE}(\pi) = \limsup_{n\to\infty} \frac{1}{n}\sum_{i=1}^{n} EL(\pi(i))$$

TP "refines" MP

- If $\mathcal{P}_1$ (minimizer) can ensure $\underline{MP} = \overline{MP} < 0$ (memoryless), he can ensure $\underline{TP} = \overline{TP} = -\infty$.
- If $\mathcal{P}_2$ (maximizer) can ensure $\underline{MP} = \overline{MP} > 0$ (memoryless), he can ensure $\underline{TP} = \overline{TP} = \infty$.

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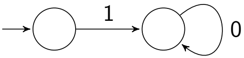
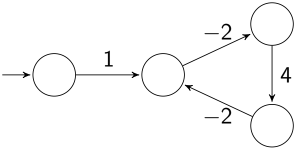
$\Rightarrow$ **TP discriminates "MP-zero" strategies** depending on the high points ($\overline{TP}$) or low points ($\underline{TP}$) of cycles.





$\overline{MP} = \underline{MP} = 0$

$\overline{TP} = \underline{TP} = 1$

$\overline{MP} = \underline{MP} = 0$

$\overline{TP} = 3, \ \underline{TP} = -1$

AE "refines" TP

AE describes the **long-run average** *EL*

$\hookrightarrow$ By definition, $\underline{AE}(\pi), \overline{AE}(\pi) \in [\underline{TP}(\pi), \overline{TP}(\pi)]$.

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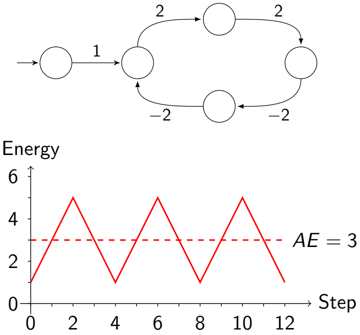
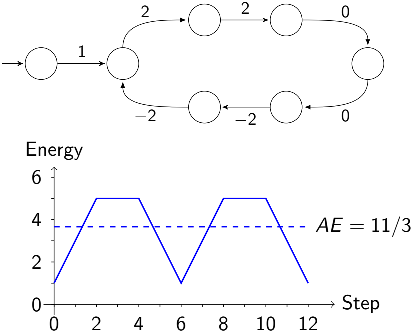
⇒ **AE discriminates strategies with identical high/low points**.





Identical MP and TP, but AE lower in the first one.

Memoryless determinacy (1/2)

Classical criteria from the literature **cannot be applied out-of-the-box** [EM79, BSV04, AR14, GZ04, Kop06].

↪ Common approach: connect *first cycle* games and infinite-duration ones.

↪ Requires e.g., closure under *cyclic permutation* and *concatenation* [AR14].

Intuitively: ability to **mix and shuffle** good cycles and stay good.

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Not true in general for AE!

$$\mathcal{C}_1 = \{-1\},\ \mathcal{C}_2 = \{1\},\ \mathcal{C}_3 = \{1, -2\}$$

$$AE(\mathcal{C}_1\mathcal{C}_2) = (-1+0)/2 = -1/2 < AE(\mathcal{C}_2\mathcal{C}_1) = (1-0)/2 = 1/2$$

$$AE(\mathcal{C}_3) = 0 \text{ but } AE(\mathcal{C}_3\mathcal{C}_3) = -1/2 < 0$$

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Intuitively: ability to **mix and shuffle** good cycles and stay good.

We can only shuffle/repeat cycles that are neutral w.r.t. the energy level!

$\hookrightarrow$ **zero-cycles**

Memoryless determinacy (2/2)

Two key properties:

1. **Extraction of prefixes**
   - ▷ Let $\rho \in \mathit{Prefs}(G)$, $\pi \in \mathit{Plays}(G)$. Then,

$$\overline{AE}(\rho \cdot \pi) = EL(\rho) + \overline{AE}(\pi).$$

2. **Extraction of a best cycle**
   - ▷ Given an infinite sequence of *zero-cycles*, one can select and repeat a *best cycle* to minimize the average-energy.

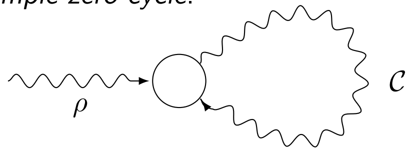
One-player games: strategy

Sketch (minimizer)

1. If you can ensure $MP < 0$, do it.
   - ▷ Memoryless [EM79], implies $AE = -\infty$.
2. If you *cannot* ensure $MP = 0$, forget it.
   - ▷ You are doomed, $AE = \infty$.
3. Play the strategy that minimizes

$$\overline{AE}(\rho \cdot \mathcal{C}^{\omega}) = EL(\rho) + \overline{AE}(\mathcal{C}),$$

where $\mathcal{C}$ is a *simple zero-cycle*.



↪ Picking the best combination can be done **without memory**.

One-player games: P algorithm (1/2)

- Case $MP < 0$ is easy
  - ▷ Look for a negative cycle (e.g., Bellman-Ford, $\mathcal{O}(|S|^3)$)

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  - ▷ Look for a negative cycle (e.g., Bellman-Ford, $\mathcal{O}(|S|^3)$)
- Assume $MP = 0$: **pick the best combination** of ρ and $\mathcal{C}$
  - ▷ Computing the best ρ for each state is easy with classical graph algorithms (e.g., Bellman-Ford).
  - ▷ **Main task**: computing the best $\mathcal{C}$ (AE-wise) for each state.

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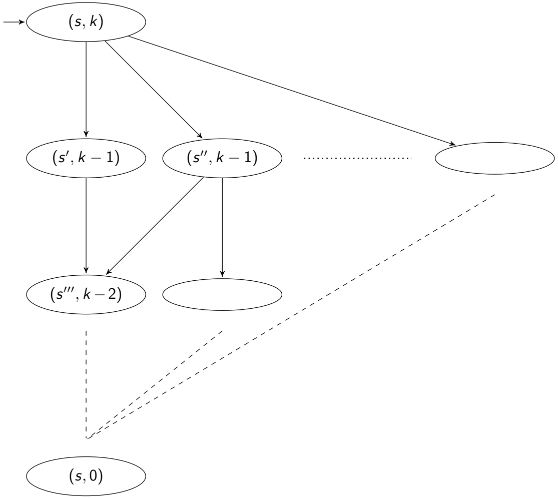
- Assume $MP = 0$: **pick the best combination** of ρ and $\mathcal{C}$
  - ▷ Computing the best ρ for each state is easy with classical graph algorithms (e.g., Bellman-Ford).
  - ▷ **Main task**: computing the best $\mathcal{C}$ (AE-wise) for each state.
- For each state, we compute the best cycle of length k, for all $k \in \{1, \ldots, |S|\}$, then pick the best one.
  - ▷ Need to compute $\mathcal{C}_{s,k}$ in polynomial time.

One-player games: P algorithm (2/2)

Computing $\mathcal{C}_{s,k}$: build a **new graph** $\mathcal{G}_{s,k}$ of size $|S| \cdot (k+1)$.



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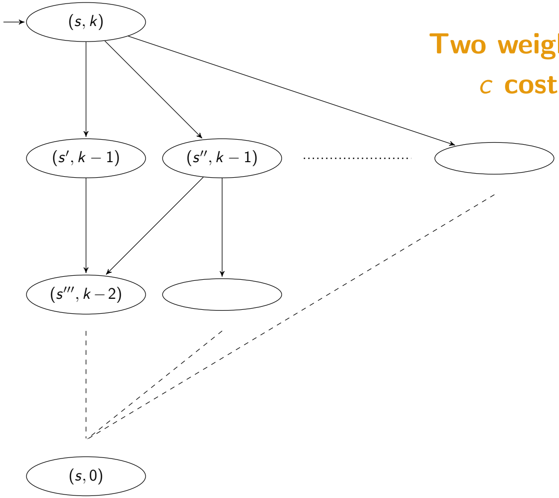

Two weights by edge: $(c, l \cdot c)$

c **cost,** l **origin level**

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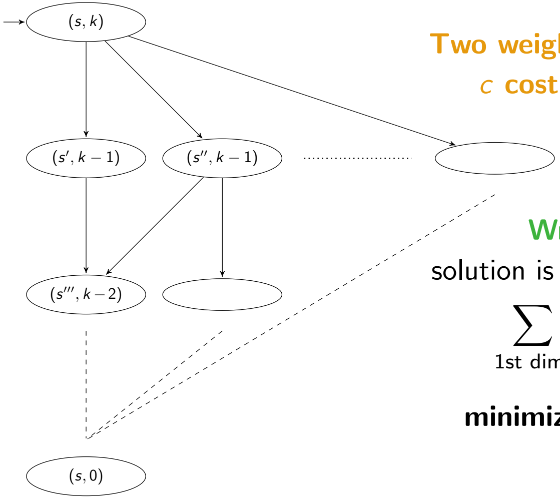

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Write an LP s.t.

solution is a path from (s,k) to $(s,0)$

$$\sum_{\text{1st dim.}} = 0 \text{ (zero cycle)}$$

$$\textbf{minimize} \sum_{\text{2nd dim.}} = AE(\mathcal{C}) \cdot k$$

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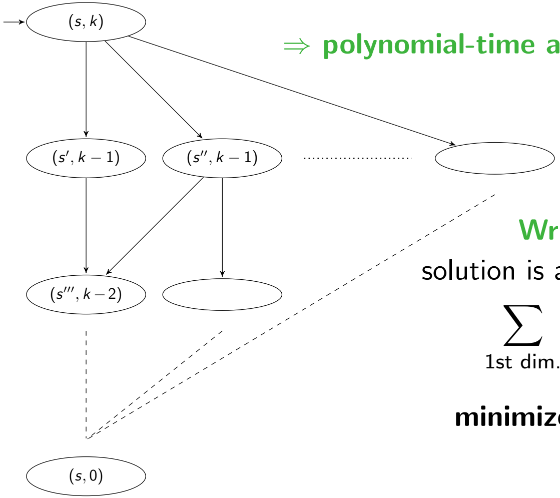

$\Rightarrow$ **polynomial-time algorithm for 1-p. games**

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$$\textbf{minimize} \sum_{\text{2nd dim.}} = AE(\mathcal{C}) \cdot k$$

Two-player games

- **Memoryless determinacy**
  - ▷ Follows from the 1-p. results (minimizer *and* maximizer) using Gimbert and Zielonka [GZ05].
- **Threshold problem in NP ∩ coNP**.
  - ▷ Memoryless determinacy + P for one-player games.
- **"Mean-payoff" hard**.
  - ▷ Replace any edge of weight c by two consecutive edges of values $2 \cdot c$ and $-2 \cdot c$.
  - ▷ Use decomposition techniques.
  - ▷ $MP(\pi)$ in $G = AE(\pi)$ in G'.

Wrap-up

Objective	**1-player**	**2-player**	**memory**
MP	P [Kar78]	NP ∩ coNP [ZP96]	memoryless [EM79]
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EG_L	P [BFL+08]	NP ∩ coNP [CdAHS03, BFL+08]	memoryless [CdAHS03]
EG_{LU}	PSPACE-c. [FJ13]	EXPTIME-c. [BFL+08]	pseudo-polynomial
AE	P	NP ∩ coNP	memoryless

▷ For all objectives but EG_{LU}, *memoryless* strategies suffice for both players.

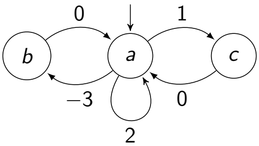
Two settings

1. AE_{LU}: AE with lower (0) and upper ($U \in \mathbb{N}$) bounds.
2. AE_L: AE with only the lower bound (0).

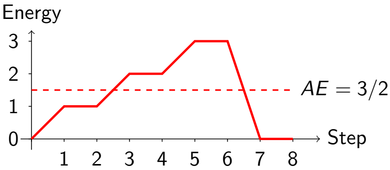
$\hookrightarrow$ Fixed initial credit $c_{\text{init}} = 0$.

Memory is needed!

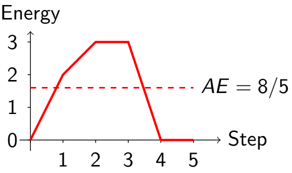
Example: $AE_{LU} \rightsquigarrow$ minimize AE while keeping $EL \in [0, 3]$.



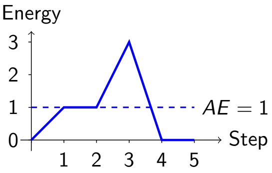
(a) One-player AE_{LU} game.



(b) Play $\pi_1 = (acacacab)^\omega$.



(c) Play $\pi_2 = (aacab)^\omega$.



(d) Play $\pi_3 = (acaab)^\omega$.

Minimal AE with π_3: alternating between the $+1$, $+2$ and -3 cycles.

Memory is needed!

Example: $AE_{LU} \rightsquigarrow$ minimize AE while keeping $EL \in [0, 3]$.

Non-trivial behavior in general!
$\hookrightarrow$ **Need to choose carefully which cycles to play.**

Memory is needed!

Example: $AE_{LU} \rightsquigarrow$ minimize AE while keeping $EL \in [0, 3]$.

Non-trivial behavior in general!
$\hookrightarrow$ Need to choose carefully which cycles to play.

The AE_{LU} problem is EXPTIME-complete.

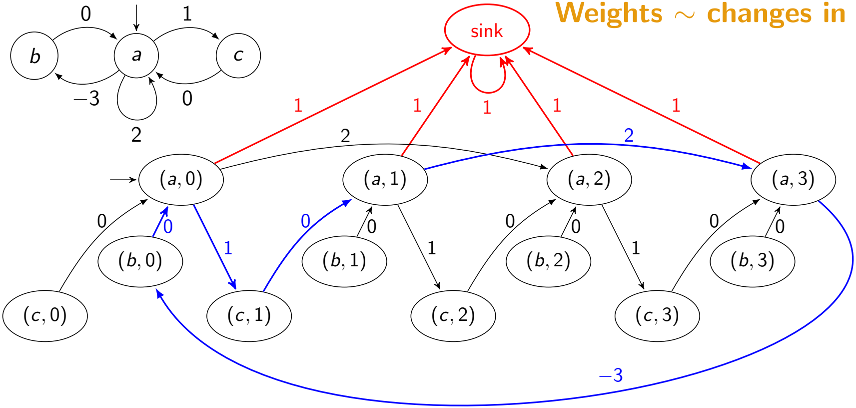
$\hookrightarrow$ Reduction from AE_{LU} to AE on pseudo-polynomial game ($\Rightarrow$ $AE_{LU} \in$ NEXPTIME $\cap$ coNEXPTIME).

$\hookrightarrow$ Reduction from this AE game to MP game + pseudo-poly. algorithm.

AE_{LU} problem: reduction to AE

- ↪ Expanded graph constraining the game within the energy bounds $[0, U]$. **Pseudo-polynomial size**: $\mathcal{O}(|S| \cdot (U + 1))$.
  - ↪ If we go out, $AE = \infty$.

Weights ∼ changes in *EL*

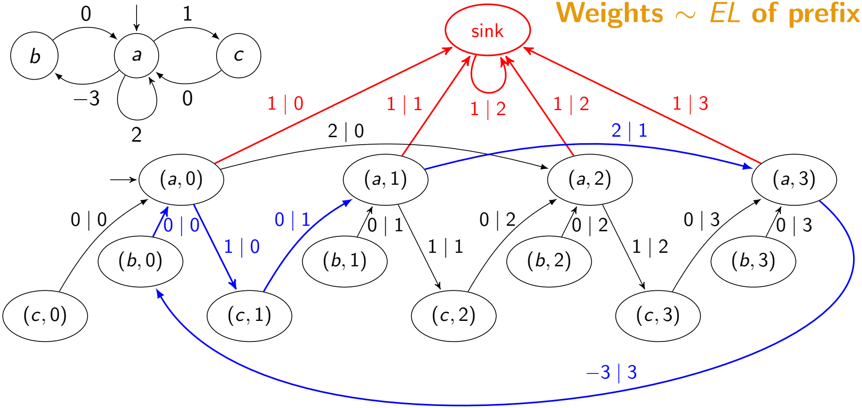


minimal $AE \wedge EL \in [0, 3]$ in $G \iff$ minimal AE in G'

AE_{LU} problem: further reduction to MP

↪ Expanded graph of pseudo-poly. size: $\mathcal{O}(|S| \cdot (U+1))$.
Threshold for AE: $t = 1$.

↪ If we go out, $MP = \lceil t \rceil + 1 > t \Rightarrow$ losing.



If $\neg(\Diamond \text{sink})$: $\overline{AE}(\pi)$ in $G' = \overline{MP}(\pi)$ in G''

AE_{LU} problem: complexity

Objective	**1-player**	**2-player**	**memory**
MP	P [Kar78]	NP ∩ coNP [ZP96]	memoryless [EM79]
TP	P [FV97]	NP ∩ coNP [GS09]	memoryless [GZ04]
EG_L	P [BFL+08]	NP ∩ coNP [CdAHS03, BFL+08]	memoryless [CdAHS03]
EG_{LU}	PSPACE-c. [FJ13]	EXPTIME-c. [BFL+08]	pseudo-polynomial
AE	P	NP ∩ coNP	memoryless
AE_{LU} (poly. U)	P	NP ∩ coNP	polynomial
AE_{LU} (arbitrary)	EXPTIME-e./PSPACE-h.	EXPTIME-c.	pseudo-polynomial

- ▷ Pseudo-poly. algo. to solve the MP problem (e.g., [BCD+11]).
- ▷ Lower bounds follow from EG_{LU}.
- ▷ Pseudo-polynomial memory is both necessary and sufficient.

AE_L problem: partial answers

- **One-player games.**
  - ↪ **Key argument:** upper-bounding the value of the energy over a witness winning path.
  - ↪ Pseudo-polynomial bound for U, then **reduction to an AE_{LU} problem.**
  - ↪ EXPTIME-algorithm.
  - ↪ **Lower bound:** NP-hard via *subset sum problem* [GJ79].

AE_L problem: partial answers

- **One-player games.**
  - ↪ **Key argument:** upper-bounding the value of the energy over a witness winning path.
  - ↪ Pseudo-polynomial bound for U, then **reduction to an AE_{LU} problem.**
  - ↪ EXPTIME-algorithm.
  - ↪ **Lower bound:** NP-hard via *subset sum problem* [GJ79].

- **Two-player games.**
  - ↪ Decidability is open.
  - ↪ **Lower bound:** EXPTIME-hard via *countdown games* [JSL08].

AE_L problem: complexity

Objective	**1-player**	**2-player**	**memory**
MP	P [Kar78]	NP ∩ coNP [ZP96]	memoryless [EM79]
TP	P [FV97]	NP ∩ coNP [GS09]	memoryless [GZ04]
EG_L	P [BFL+08]	NP ∩ coNP [CdAHS03, BFL+08]	memoryless [CdAHS03]
EG_{LU}	PSPACE-c. [FJ13]	EXPTIME-c. [BFL+08]	pseudo-polynomial
AE	P	NP ∩ coNP	memoryless
AE_{LU} (poly. U)	P	NP ∩ coNP	polynomial
AE_{LU} (arbitrary)	EXPTIME-e./PSPACE-h.	EXPTIME-c.	pseudo-polynomial
AE_L	EXPTIME-e./NP-h.	*open*/EXPTIME-h.	*open* (≥ pseudo-poly.)

Wrap-up

"New" quantitative objective.

- Practical motivations (e.g., HYDAC).
- "Refines" TP (and MP).
- Same complexity class as EG_L, MP and TP games.
- AE_{LU} well-understood.
- Open questions for AE_L.

Thank you! Any question?

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