# Looking at Mean-Payoff and Total-Payoff through Windows

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# Aim of this talk

1. Overview of the situation for (multi) MP and TP games
   - ▷ No P algorithm known in one dimension
   - ▷ In multi dimensions, MP is coNP-complete
   - ▷ First contribution: **TP is undecidable in multi dimensions**
   - ▷ No timing guarantee

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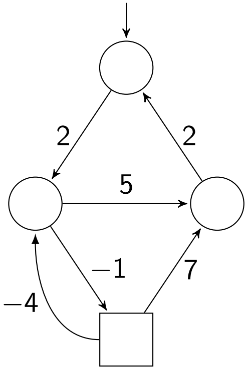
1. Overview of the situation for (multi) MP and TP games
   - ▷ No P algorithm known in one dimension
   - ▷ In multi dimensions, MP is coNP-complete
   - ▷ First contribution: **TP is undecidable in multi dimensions**
   - ▷ No timing guarantee

2. Introduction of **window objectives**
   - ▷ Conservative approximation of MP/TP
   - ▷ Break the complexity barriers
   - ▷ Specifies timing requirements
   - ▷ Algorithms, complexity and memory requirements
   - ▷ Several flavors of the objective

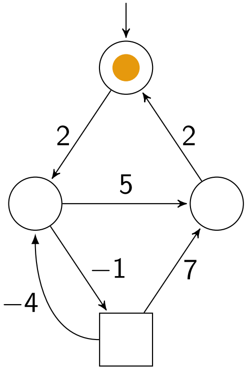
Full details available on arXiv: abs/1302.4248

# MP and TP games




- $G = (S_1, S_2, E, w)$
- $S = S_1 \cup S_2, S_1 \cap S_2 = \emptyset, E \subseteq S \times S$, $w \colon E \to \mathbb{Z}$
- $\mathcal{P}_1$ states $= \bigcirc$
- $\mathcal{P}_2$ states $= \square$
- Plays, prefixes, **pure** strategies.

# MP and TP games




- $\mathrm{TP}(\pi) = \liminf_{n\to\infty} \sum_{i=0}^{i=n-1} w(s_i, s_{i+1})$
- $\mathrm{MP}(\pi) = \liminf_{n\to\infty} \frac{1}{n} \mathrm{TP}(\pi(n))$

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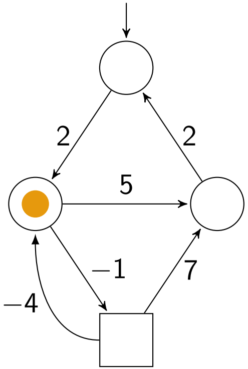


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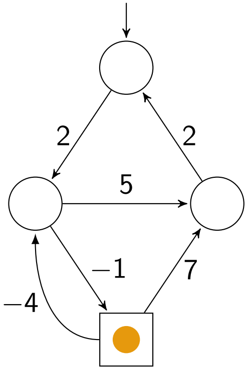


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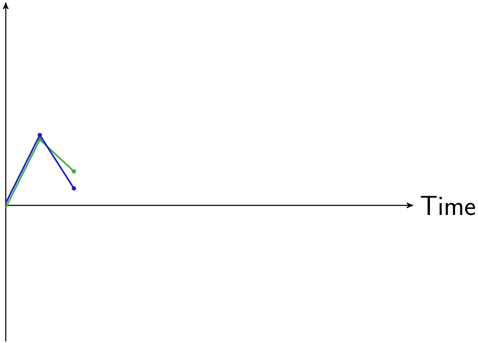

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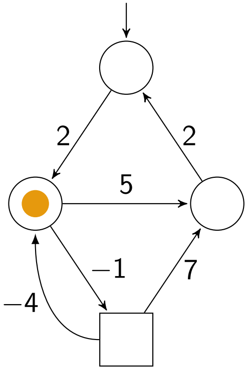


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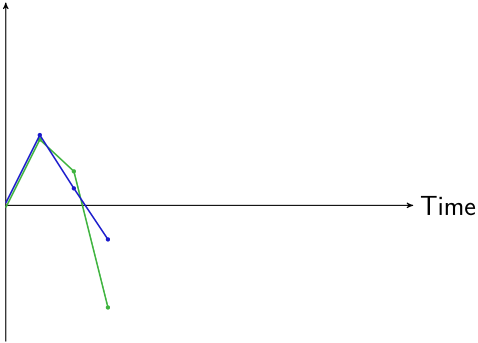

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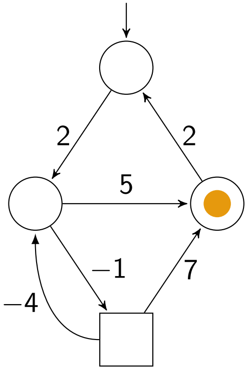


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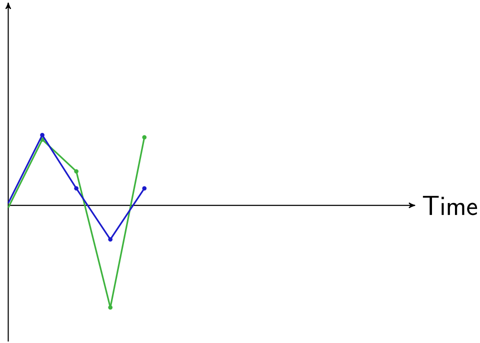

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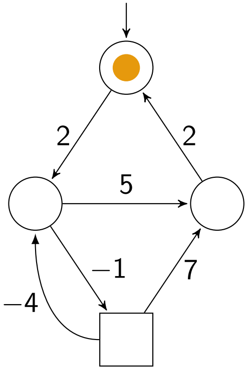


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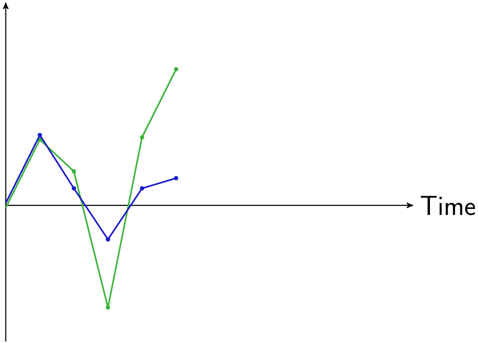

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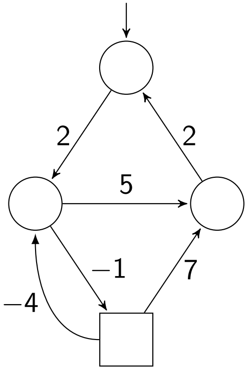


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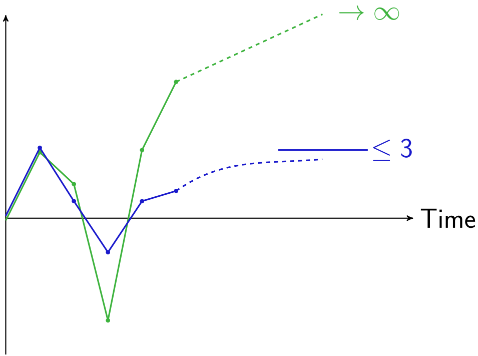

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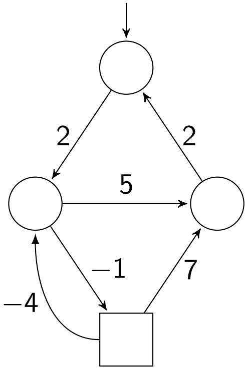


Then, $(2, 5, 2)^\omega$

- **TP (MP) threshold problem**

  Given $v \in \mathbb{Q}$ and $s_{\mathsf{init}} \in S$,

  $\exists? \, \lambda_1 \in \Lambda_1$ s.t. $\forall \lambda_2 \in \Lambda_2$,

  $\underline{\mathsf{TP}}(\mathsf{Outcome}_G(s_{\mathsf{init}}, \lambda_1, \lambda_2)) \geq v$

# Known results

| | one-dimension | | | $k$-dimension | | |
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| | complexity | $\mathcal{P}_1$ mem. | $\mathcal{P}_2$ mem. | complexity | $\mathcal{P}_1$ mem. | $\mathcal{P}_2$ mem. |
| $\underline{\mathsf{MP}}$ / $\overline{\mathsf{MP}}$ | NP $\cap$ coNP | mem-less | | coNP-c. / NP $\cap$ coNP | infinite | mem-less |
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- Long tradition of study. Non-exhaustive selection: [EM79, ZP96, Jur98, GZ04, GS09, CDHR10, VR11, CRR12]
- $k$-dimension: weights = integer vectors
- *No known polynomial time algorithm for one-dimension*
- *No result on multi-dimension total-payoff*

# Multi-dimension TP games are undecidable

**Theorem**

The threshold problem for infimum and supremum total-payoff objectives is **undecidable** in multi-dimension games, for five dimensions.

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- ▷ Reduction from the **halting problem for 2CMs** [Min61]

# Two-counter machines

- Finite set of instructions
- Two counters $C_1$ and $C_2$ taking values $(v_1, v_2) \in \mathbb{N}^2$
- Instructions:
  - ▷ Increment

$$C_i + +$$

  - ▷ Decrement

$$C_i - -$$

  - ▷ Zero test and branch accordingly

$$\textbf{If } C_i == 0 \textbf{ do } \textit{this} \textbf{ else do } \textit{that}$$

- W.l.o.g. if the machine stops, it stops with both counters to zero

# Encoding a 2CM in a 5-dim. TP game

- TP objective (inf or sup) of threshold $(0, 0, 0, 0, 0)$
- $\mathcal{P}_1$ must simulate faithfully
- $\mathcal{P}_2$ retaliates if $\mathcal{P}_1$ cheats
- At the end, $\mathcal{P}_1$ wins the TP game **iff** the 2CM stops

**Key idea**: after $m$ steps, the TP has value $(v_1, -v_1, v_2, -v_2, -m)$ **iff** the 2CM counters have value $(v_1, v_2)$

# Instructions

- Increment $C_1$

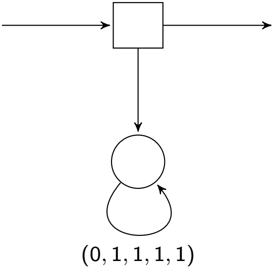
$$\bigcirc \xrightarrow{(1,-1,0,0,-1)} \bigcirc$$

- Decrement $C_1$

$$\bigcirc \xrightarrow{(-1,1,0,0,-1)} \bigcirc$$

# Instructions

- Checking counter $C_1$ is non-negative

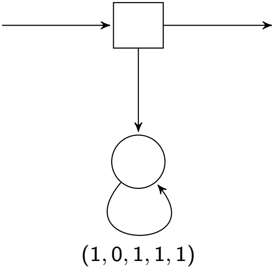



▷ If $\mathcal{P}_1$ cheats, he is doomed!

▷ Otherwise, $\mathcal{P}_2$ has no interest in retaliating.

# Instructions

- Checking a zero test on $C_1$




- ▷ If $\mathcal{P}_1$ cheats, he is doomed!
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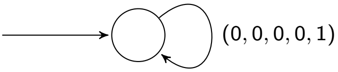
# Halting

- If the 2CM halts (with counters to zero w.l.o.g.)




▷ Thanks to the fifth dim., $\mathcal{P}_1$ wins only if the machine halts.

# The case is closed

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| $\underline{\text{TP}}$ / $\overline{\text{TP}}$ | NP $\cap$ coNP | mem-less | | **Undec.** | - | - |

# Motivations

- Classical MP and TP objectives have some drawbacks
  - Complexity issues
    - P membership for the one-dim. case is a long-standing open problem
    - TP undecidable in $k$-dim.
  - Infimum vs. supremum
  - **no timing guarantee**: the "good behavior" occurs at the limit...

# Window objectives: key idea

- **Window** of fixed size **sliding** along a play
  $\rightsquigarrow$ defines a local finite horizon
- Objective: see a **local** $MP \geq 0$ *before hitting the end* of the window
  $\rightsquigarrow$ needs to be verified at *every* step

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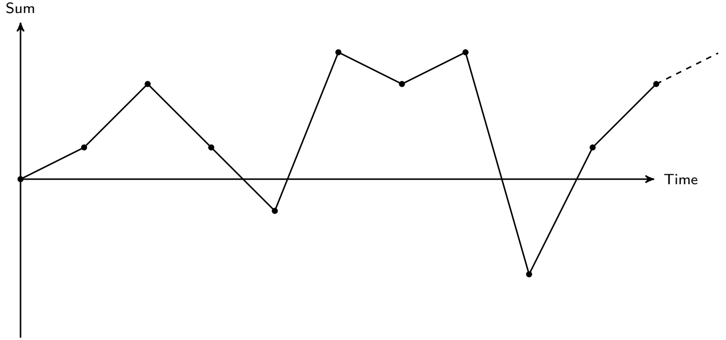
▷ Conservative approximation of MP/TP

▷ Intuition: local deviations from the threshold must be compensated in a parametrized # of steps

▷ Variety of results and algorithms

# Illustration: WMP, threshold zero, maximal window = 4



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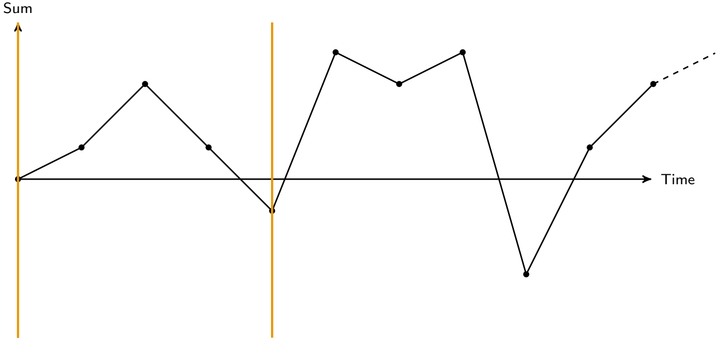

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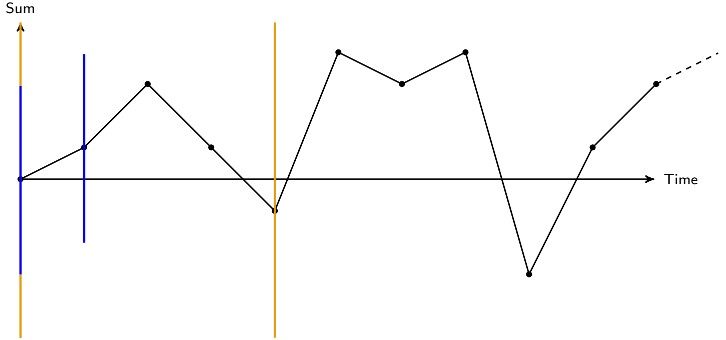

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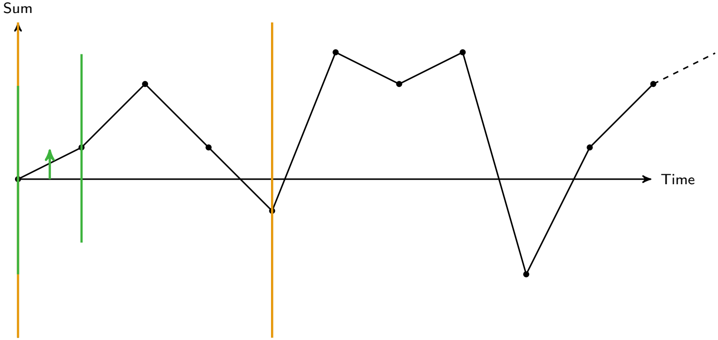

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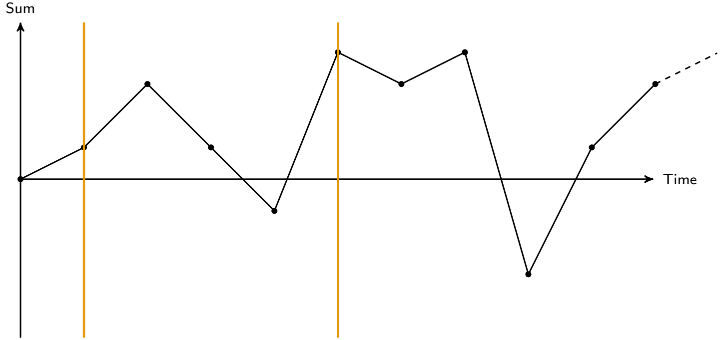

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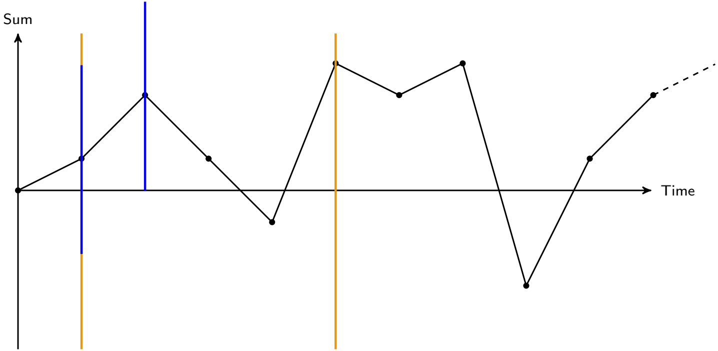

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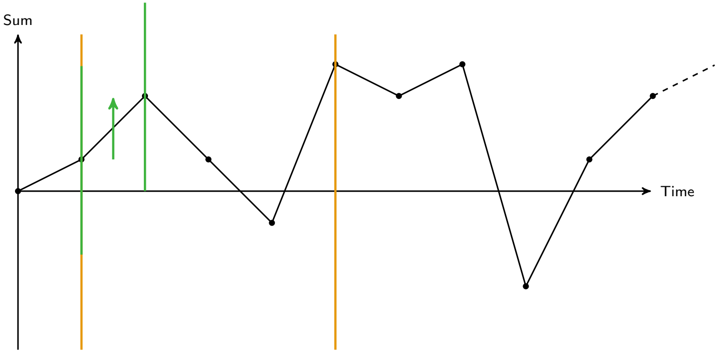

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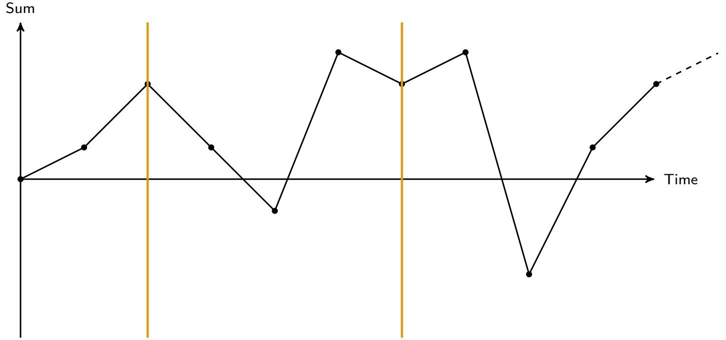

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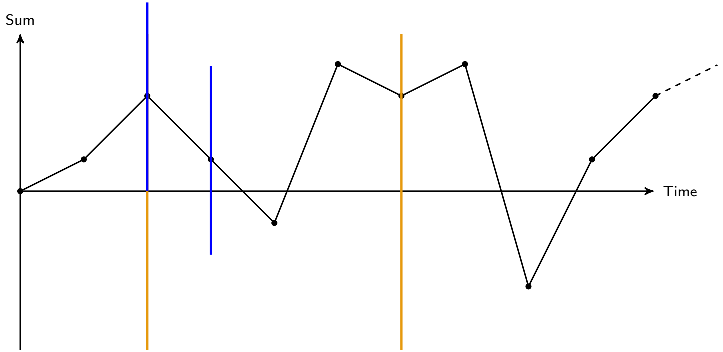

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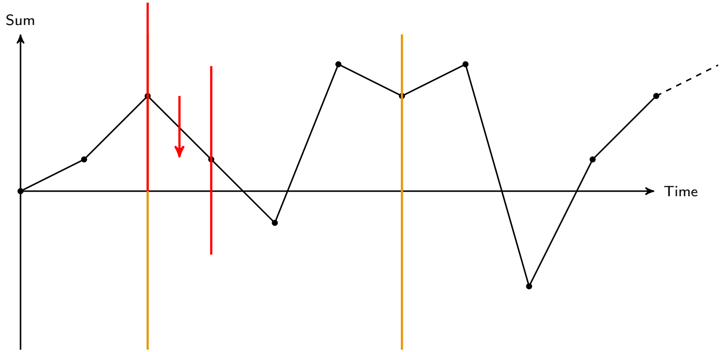

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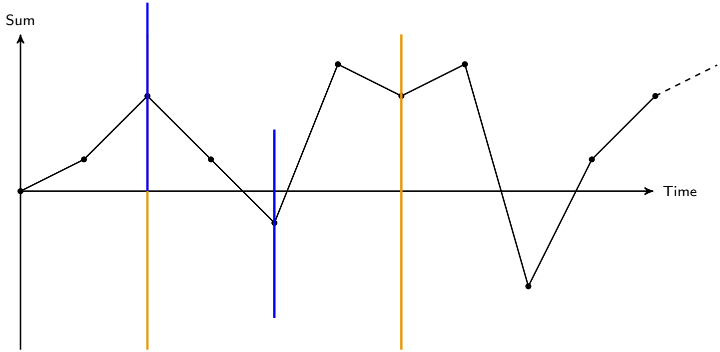

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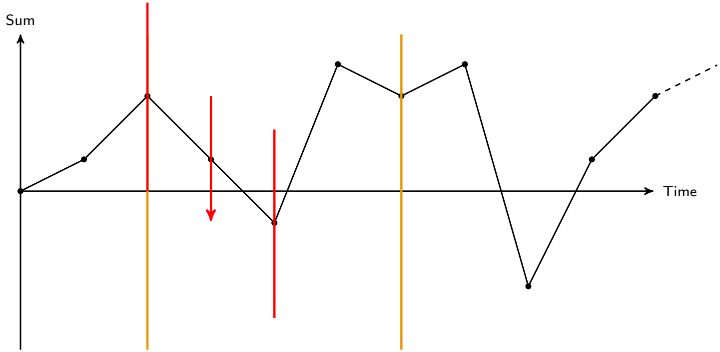

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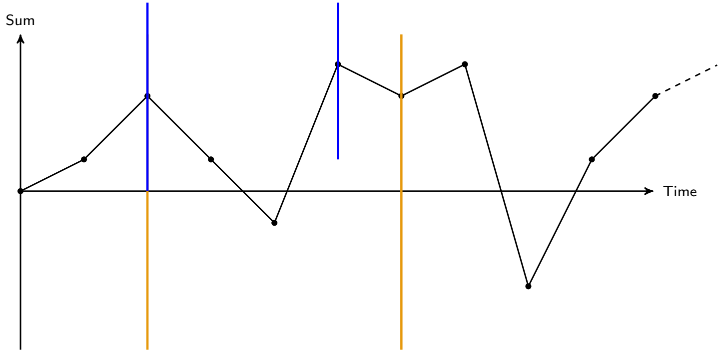

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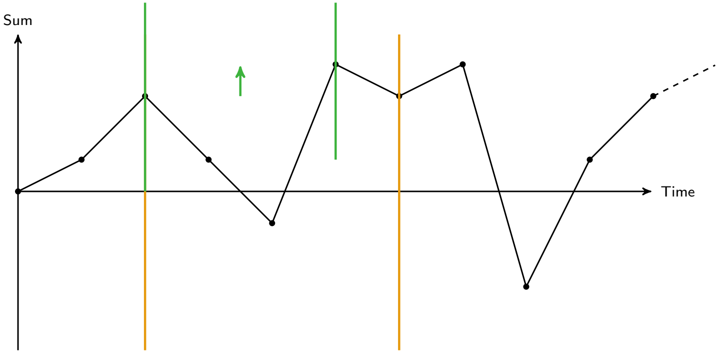

# Multiple variants

- Given $l_{\max} \in \mathbb{N}_0$, *good window* $\mathbf{GW}(l_{\max})$ asks for a positive sum in at most $l_{\max}$ steps (one window, from the first state)
- *Direct Fixed Window*: $\mathbf{DFW}(l_{\max}) \equiv \Box\mathbf{GW}(l_{\max})$
- *Fixed Window*: $\mathbf{FW}(l_{\max}) \equiv \Diamond\mathbf{DFW}(l_{\max})$
- *Direct Bounded Window*: $\mathbf{DBW} \equiv \exists\, l_{\max},\ \mathbf{DFW}(l_{\max})$
- *Bounded Window*: $\mathbf{BW} \equiv \Diamond\mathbf{DBW} \equiv \exists\, l_{\max},\ \mathbf{FW}(l_{\max})$

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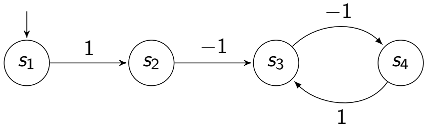
▷ A window *closes* when the sum becomes positive

▷ A window is *open* if not yet closed

# Examples




- **FW**(2) is satisfied, **DBW** is not, MP is satisfied.

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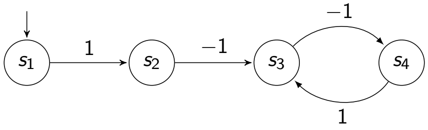


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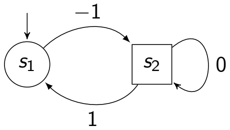


- MP is satisfied but none of the window objectives is.

# Conservative approximation of MP (one-dim.)

**The following are true**

$$\text{Any window obj.} \Rightarrow \textbf{BW} \Rightarrow \text{MP} \geq 0$$
$$\textbf{BW} \Leftarrow \text{MP} > 0$$

# Results overview

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| WMP: fixed polynomial window | **P-c.** | **mem. req.** $\leq$ **linear**($\lvert S\rvert \cdot l_{\max}$) | | **PSPACE-h.** **EXP-easy** | **exponential** | |
| WMP: fixed arbitrary window | **P**($\lvert S\rvert, V, l_{\max}$) | | | **EXP-c.** | | |
| WMP: bounded window problem | **NP $\cap$ coNP** | **mem-less** | **infinite** | **NPR-h.** | - | - |

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- $|S|$ the # of states, $V$ the length of the binary encoding of weights, and $l_{\max}$ the window size.
- For one-dim. games with poly. windows, we are in **P**
- For multi-dim. games with fixed windows, we are **decidable**
- Window obj. provide **timing guarantees**

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- $\lvert S\rvert$ the # of states, $V$ the length of the binary encoding of weights, and $l_{\max}$ the window size.

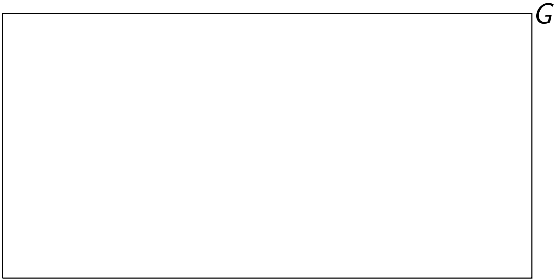
- No time to discuss everything. Focus.

# High level sketch: top-down approach

- $\mathbf{FW}(l_{\max}) \equiv \Diamond\mathbf{DFW}(l_{\max})$

▷ Assume we can compute $\mathbf{DFW}(l_{\max})$,

▷ Compute attractor, declare winning and recurse on subgame.



# High level sketch: top-down approach

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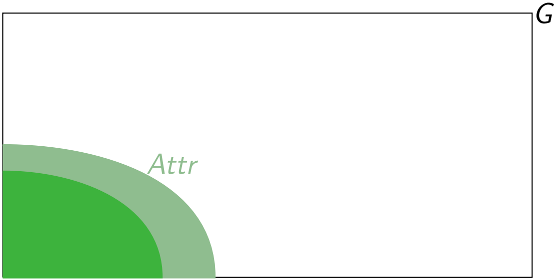

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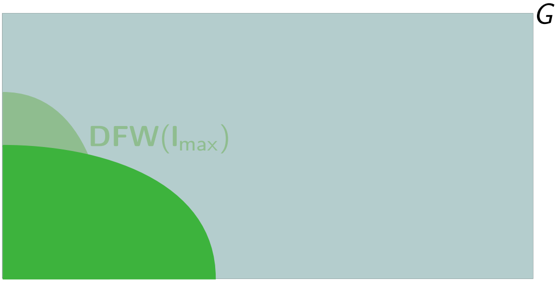

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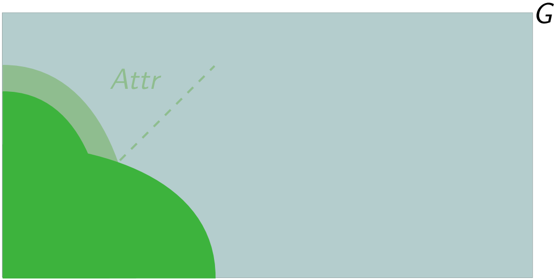

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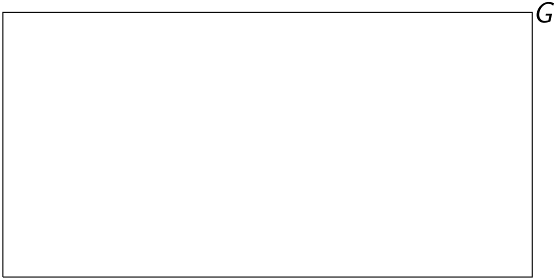

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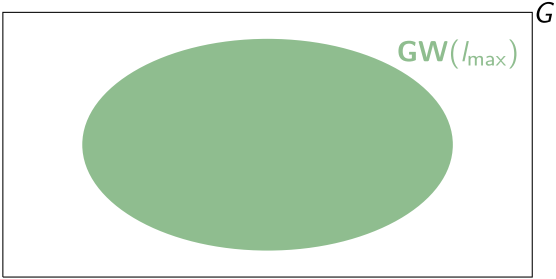

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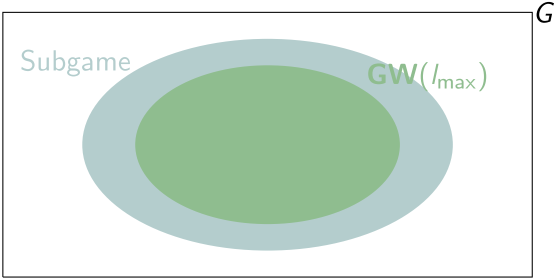

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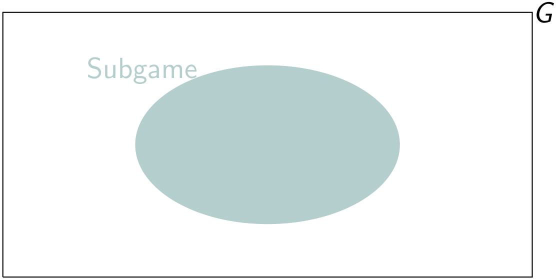

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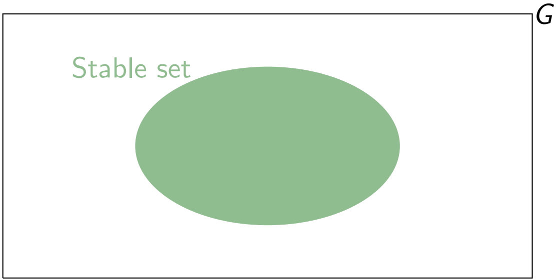

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▷ Simply compute the best sum achievable in at most $l_{\max}$ steps and check if positive.

- Finally,

**Theorem**

In two-player one-dimension games,
(a) the fixed arbitrary window MP problem is decidable in time polynomial in the size of the game and the window size,
(b) **the fixed polynomial window MP problem is P-complete**,
(c) both players require memory, and memory of size linear in the size of the game and the window size is sufficient.

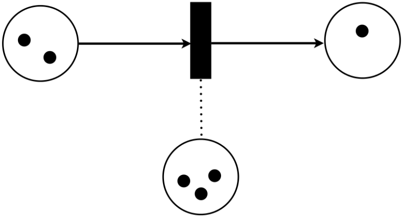
# Approach

- We prove **non-primitive recursive**[1] **(NPR) hardness**
- Reduction from the termination problem in **reset nets** (Petri nets with reset arcs) [Sch02]

[1] Cf. Ackermann function

# Reset nets

- Classic Petri net (places, tokens, transitions) with added *reset arcs*



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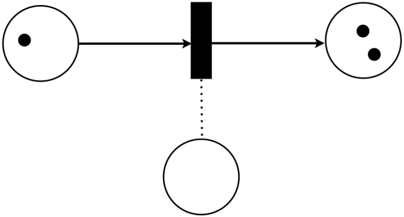

▷ Transitions may empty a place from all its tokens

▷ Given an initial marking, the *termination problem* asks if there exists an infinite sequence of transitions that can be fired

# From reset nets to **direct** bounded window games

- Crux of the construction: encoding the markings
  - We use one dimension for each place
  - If a place $p$ contains $m$ tokens, then there will be an open window on dimension $p$ with sum value $-m-1$
  - Hence **during a faithful simulation, all windows remain open** (you cannot consume tokens that do not exist)

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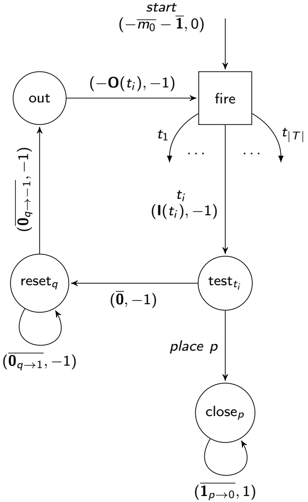
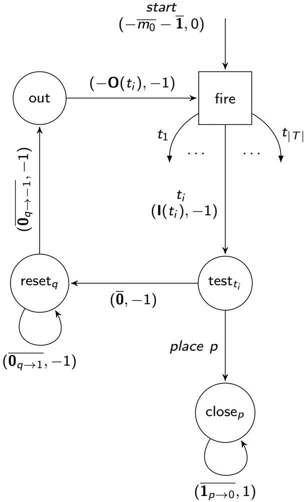
- $\mathcal{P}_2$ simulates the net
- $\mathcal{P}_1$ checks if he is faithful
- $\mathcal{P}_1$ wants to win the direct bounded window MP obj.
  - only able to do so if $\mathcal{P}_2$ cheats, i.e., if all runs terminate

# The construction in a nutshell



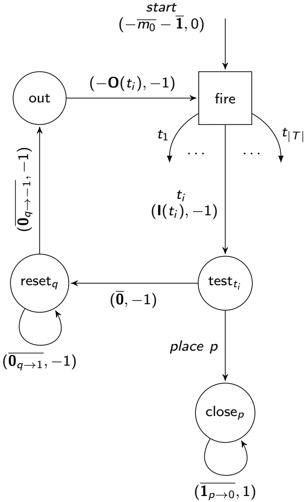

- ▷ The initial marking open corresponding windows in all places
- ▷ $\mathcal{P}_2$ chooses transitions to fire, which consume tokens
- ▷ $\mathcal{P}_1$ can branch or continue (and apply reset, then output)

# The construction in a nutshell




- If no infinite execution exists, at some point, $\mathcal{P}_2$ must choose a transition without the needed tokens on some place $p$
- The window closes on dimension $p$
- By branching $\mathcal{P}_1$ can close all other windows and ensure winning

# The construction in a nutshell




- ▷ If $\mathcal{P}_1$ branches while $\mathcal{P}_2$ is honest, one window stays open forever and he loses
- ▷ The additional dimension ensures that $\mathcal{P}_1$ leaves the reset state

# Extension to bounded window objective

- More involved construction

**Theorem**

In two-player multi-dimension games, the bounded window mean-payoff problem is non-primitive recursive hard.

# A new family of objectives

| | one-dimension | | | $k$-dimension | | |
|---|---|---|---|---|---|---|
| | complexity | $\mathcal{P}_1$ mem. | $\mathcal{P}_2$ mem. | complexity | $\mathcal{P}_1$ mem. | $\mathcal{P}_2$ mem. |
| $\underline{\text{MP}}$ / $\overline{\text{MP}}$ | NP $\cap$ coNP | mem-less | | coNP-c. / NP $\cap$ coNP | infinite | mem-less |
| $\underline{\text{TP}}$ / $\overline{\text{TP}}$ | NP $\cap$ coNP | mem-less | | **undec.** | - | - |
| WMP: fixed polynomial window | **P-c.** | **mem. req.** $\leq$ **linear**$(\lvert S\rvert \cdot l_{\max})$ | | **PSPACE-h.** **EXP-easy** | **exponential** | |
| WMP: fixed arbitrary window | **P**$(\lvert S\rvert, V, l_{\max})$ | | | **EXP-c.** | | |
| WMP: bounded window problem | **NP $\cap$ coNP** | **mem-less** | **infinite** | **NPR-h.** | - | - |

- Conservative approximation of MP/TP
- Provides timing guarantees
- Breaks the NP $\cap$ coNP barrier in one-dim. poly. window case
- Decidable approximation of TP in multi-dim. case
- *Open question*: is BW decidable in multi-dim. ?

Check the full version on arXiv! abs/1302.4248

**Thanks!**

Do not hesitate to discuss with us!

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