Synthesis in Multi-Criteria Quantitative Games

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LSV - CNRS & ENS Cachan, France

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Ackermann Award Lecture

Thesis: synthesis in multi-criteria quantitative games

- ▷ University of Mons, Belgium, April 2014.
- ▷ Currently post-doc in LSV, France, with Patricia Bouyer and Nicolas Markey.

Véronique Bruyère
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Aim of this talk

Sketch the **motivation** for the research and give some **examples** of the studied problems.

General context

- Verification and synthesis:
  - ▷ a reactive **system** to *control*,
  - ▷ an *interacting* **environment**,
  - ▷ a **specification** to *enforce*.

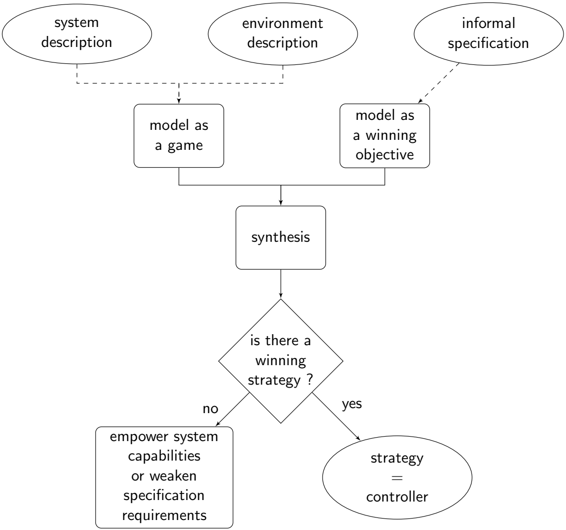
- Automated **controller synthesis** via games.

General context

- Verification and synthesis:
  - ▷ a reactive **system** to *control*,
  - ▷ an *interacting* **environment**,
  - ▷ a **specification** to *enforce*.
- Automated **controller synthesis** via games.
- Strong links between logic and games:
  - ▷ logic used as specification language,
  - ▷ model checking via game solving (e.g., parity games for modal μ-calculus).

Here: focus on the game-theoretic view.

Synthesis via two-player graph games



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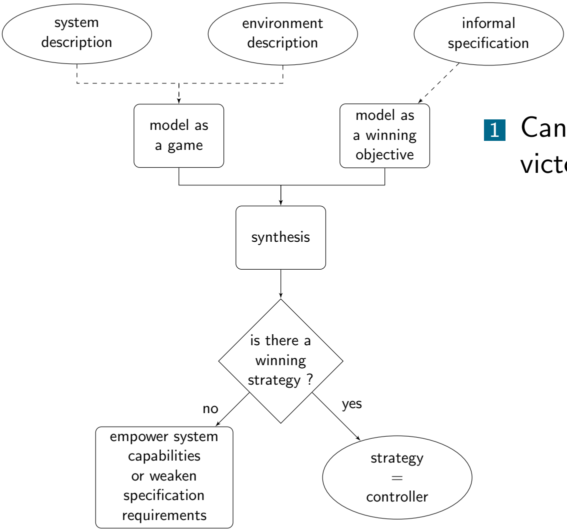
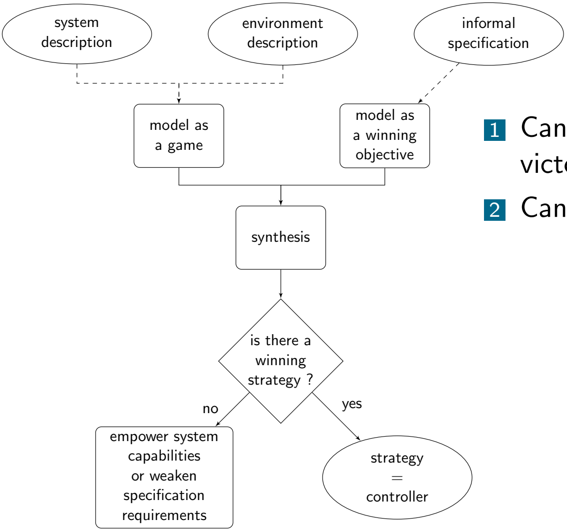


1. Can one player guarantee victory?

Synthesis via two-player graph games



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2. Can we decide which one?

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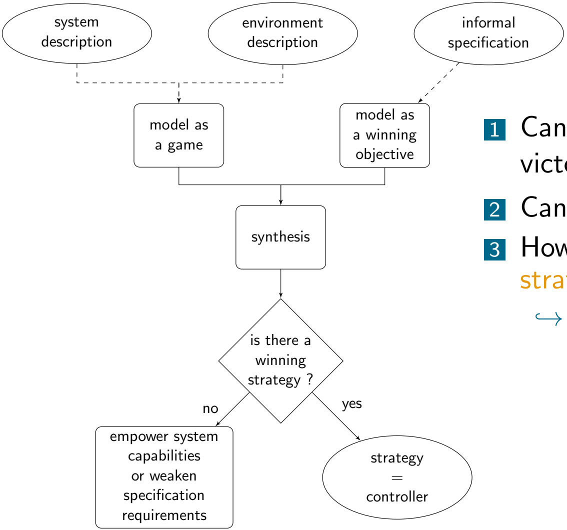
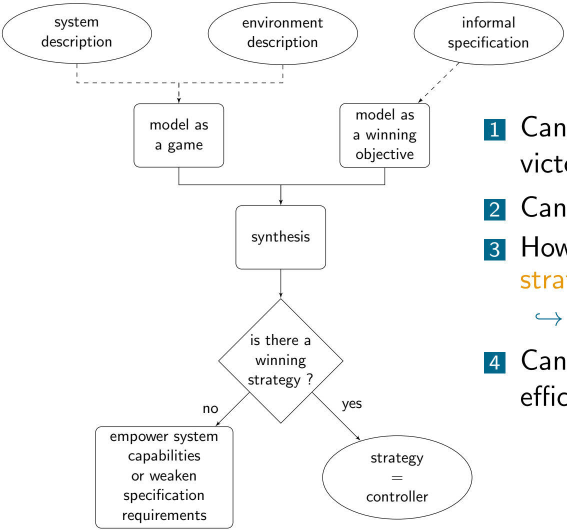


1. Can one player guarantee victory?
2. Can we decide which one?
3. How complex his winning strategy needs to be?
   $\hookrightarrow$ **Simpler is better.**

Synthesis via two-player graph games



1. Can one player guarantee victory?
2. Can we decide which one?
3. How complex his winning strategy needs to be?
   ↪ **Simpler is better.**
4. Can we synthesize one efficiently?

From Boolean to quantitative and beyond: an ongoing shift

Boolean view: behavior is either correct or incorrect.
No interpretation of *how good* it is.
OK for yes-no properties (e.g., no deadlock).
Example: parity games [GTW02, EJS93, Jur98].

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Quantitative view: rank the performance, model resource constraints.
Traditionally, only *single-criterion* models.
OK for energy consumption, response time [CdAHS03, BCHJ09, Ran13].
Example: mean-payoff games [EM79, ZP96, BCD$^+$11].

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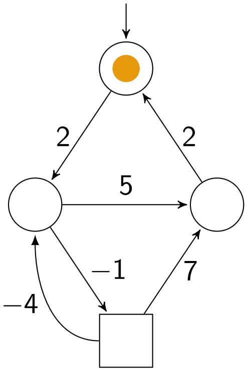
↓

Multi-criteria view: study interplays and trade-offs.
E.g., response time vs. computing power vs. energy consumption.
Also, consider strategies with richer guarantees.
E.g., average performance vs. worst-case performance.

Quantitative games on graphs



- Graph $\mathcal{G} = (S, E, w)$ with $w\colon E \to \mathbb{Z}$
- Deterministic transitions
- Two-player *game* $G = (\mathcal{G}, S_1, S_2)$
  - ▷ $\mathcal{P}_1$ states = ◯
  - ▷ $\mathcal{P}_2$ states = □
- Plays have values
  - ▷ $f\colon \text{Plays}(\mathcal{G}) \to \mathbb{R} \cup \{-\infty, \infty\}$
- Players follow *strategies*
  - ▷ $\lambda_i\colon \text{Prefs}_i(G) \to \mathcal{D}(S)$
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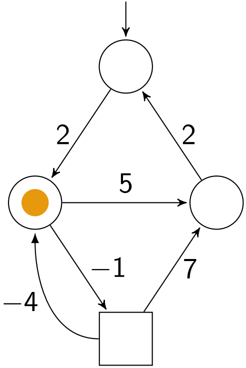

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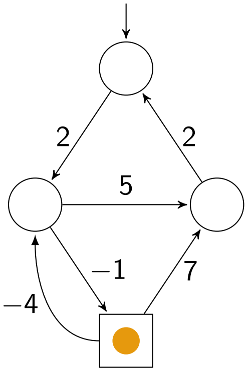

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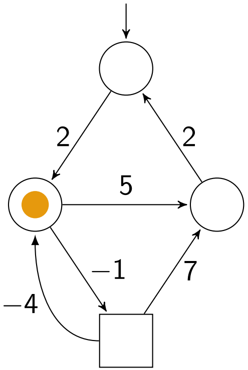

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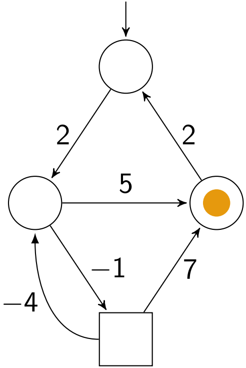

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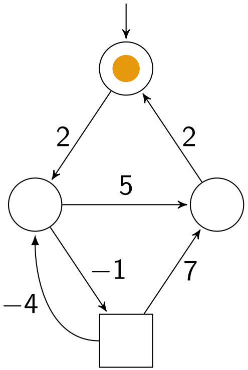

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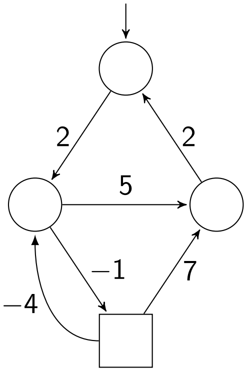

Then, $(2, 5, 2)^\omega$

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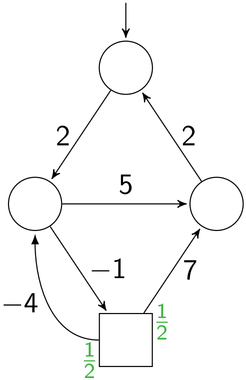
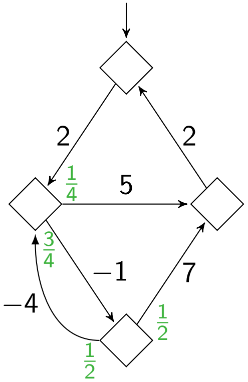
Markov decision processes



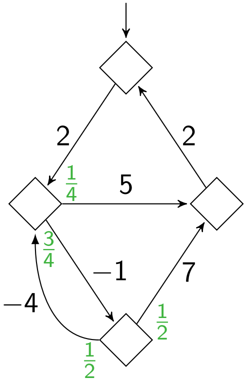
- MDP $P = (\mathcal{G}, S_1, S_\Delta, \Delta)$ with $\Delta\colon S_\Delta \to \mathcal{D}(S)$
  - ▷ $\mathcal{P}_1$ states = ◯
  - ▷ stochastic states = □
- MDP = game + strategy of $\mathcal{P}_2$
  - ▷ $P = G[\lambda_2]$

Markov chains



- MC $M = (\mathcal{G}, \delta)$ with $\delta\colon S \to \mathcal{D}(S)$
- MC $=$ MDP $+$ strategy of $\mathcal{P}_1$
  $=$ game $+$ both strategies
  - ▷ $M = P[\lambda_1] = G[\lambda_1, \lambda_2]$

Markov chains



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  $=$ game $+$ both strategies
  - ▷ $M = P[\lambda_1] = G[\lambda_1, \lambda_2]$
- Event $\mathcal{A} \subseteq \text{Plays}(\mathcal{G})$
  - ▷ probability $\mathbb{P}^M_{s_{\text{init}}}(\mathcal{A})$
- Measurable $f\colon \text{Plays}(\mathcal{G}) \to \mathbb{R} \cup \{-\infty, \infty\}$
  - ▷ *expected value* $\mathbb{E}^M_{s_{\text{init}}}(f)$

Winning semantics and decision problems

- **Qualitative** objectives - $\phi \subseteq \text{Plays}(G)$
  - ▷ λ_1 *surely winning*: $\forall\, \lambda_2 \in \Lambda_2$, $\text{Outs}_G(s_{\text{init}}, \lambda_1, \lambda_2) \subseteq \phi$
  - ▷ λ_1 *almost-surely winning*: $\forall\, \lambda_2 \in \Lambda_2$, $\mathbb{P}^{G[\lambda_1, \lambda_2]}_{s_{\text{init}}}(\phi) = 1$

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- **Quantitative** objectives - $f\colon \mathsf{Plays}(G) \rightarrow \mathbb{R} \cup \{-\infty, \infty\}$
  - ▷ *worst-case threshold problem*, $\mu \in \mathbb{Q}$:

$$\exists ?\, \lambda_1 \in \Lambda_1, \forall\, \lambda_2 \in \Lambda_2, \forall\, \pi \in \mathsf{Outs}_G(s_{\mathsf{init}}, \lambda_1, \lambda_2),\ f(\pi) \geq \mu$$

  - ▷ *expected value threshold problem* (MDP), $\nu \in \mathbb{Q}$:

$$\exists ?\, \lambda_1 \in \Lambda_1, \mathbb{E}^{P[\lambda_1]}_{s_{\mathsf{init}}}(f) \geq \nu$$

Classical qualitative objectives

- $\mathsf{Reach}_G(T) = \{\pi = s_0 s_1 s_2 \ldots \in \mathsf{Plays}(G) \mid \exists\, i \in \mathbb{N},\ s_i \in T\}$
- $\mathsf{Buchi}_G(T) = \{\pi = s_0 s_1 s_2 \ldots \in \mathsf{Plays}(G) \mid \mathsf{Inf}(\pi) \cap T \neq \emptyset\}$
- $\mathsf{Parity}_G = \{\pi = s_0 s_1 s_2 \ldots \in \mathsf{Plays}(G) \mid \mathsf{Par}(\pi) \bmod 2 = 0\}$

Classical quantitative objectives and value functions

- *Total-payoff*: $\underline{\text{TP}}(\pi) = \liminf_{n\to\infty} \sum_{i=0}^{i=n-1} w((s_i, s_{i+1}))$
- *Mean-payoff*: $\underline{\text{MP}}(\pi) = \liminf_{n\to\infty} \frac{1}{n} \sum_{i=0}^{i=n-1} w((s_i, s_{i+1}))$
- *Shortest path*: truncated sum up to first visit of $T \subseteq S$
- *Energy*: keep the running sum positive at all times

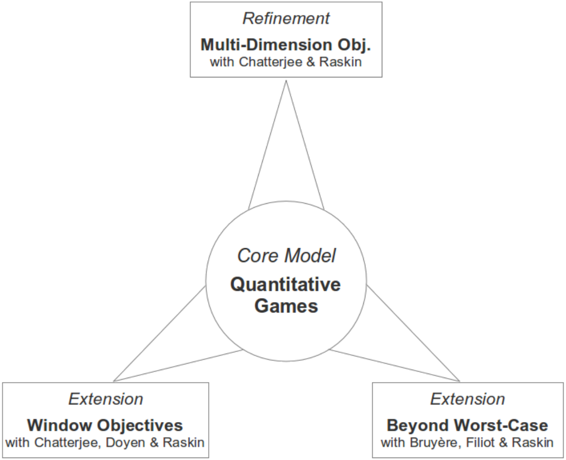
Single-criterion models - known results

		reachability	Büchi	parity
GAMES *sure sem.*	complexity	P-c.		$UP \cap coUP$
	$\mathcal{P}_1$ mem.	pure memoryless		
	$\mathcal{P}_2$ mem.			
MDPS *almost-sure sem.*	complexity	P-c.		
	$\mathcal{P}_1$ mem.	pure memoryless		

		TP	MP	SP	EG
GAMES *worst-case*	complexity	$UP \cap coUP$		P-c.	$UP \cap coUP$
	$\mathcal{P}_1$ mem.	pure memoryless			
	$\mathcal{P}_2$ mem.				
MDPS *expected value*	complexity	P-c.			n/a
	$\mathcal{P}_1$ mem.	pure memoryless			

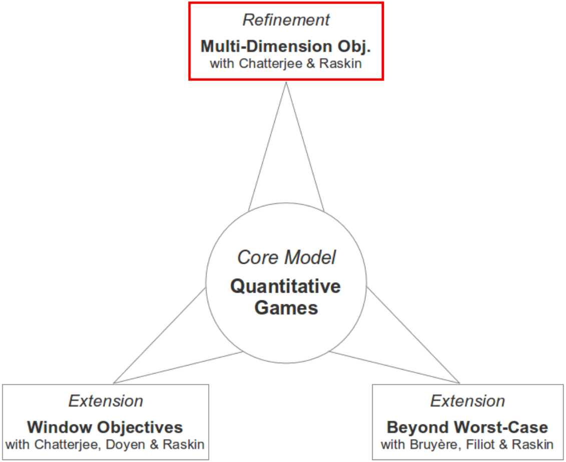
- ▷ Simple strategies suffice (no memory, no randomness).
- ▷ "Low" complexity but important open problem: $UP \cap coUP \rightsquigarrow P$?

Contributions



Shift from single-criterion models to **multi-criteria** ones.

Multi-dimension objectives



Weights = vectors $\rightsquigarrow$ multiple objectives $\rightsquigarrow$ **trade-offs**

Complexity $\nearrow$ (e.g., coNP-c. for solving energy games)

Multi-dimension objectives

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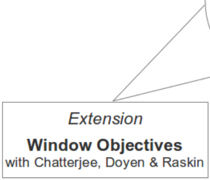
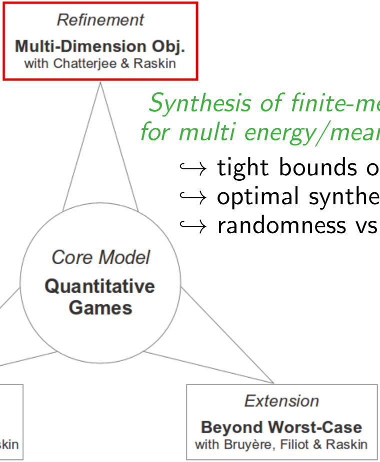

Synthesis of finite-memory strategies for multi energy/mean-payoff + parity

$\hookrightarrow$ tight bounds on memory
$\hookrightarrow$ optimal synthesis algorithm
$\hookrightarrow$ randomness vs. memory

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Refinement
Multi-Dimension Obj.
with Chatterjee & Raskin

Synthesis of finite-memory strategies for multi energy/mean-payoff + parity

- $\hookrightarrow$ tight bounds on memory
- $\hookrightarrow$ optimal synthesis algorithm
- $\hookrightarrow$ randomness vs. memory

Core Model
Quantitative Games

Undecidability of multi total-payoff

Extension
Window Objectives
with Chatterjee, Doyen & Raskin

Extension
Beyond Worst-Case
with Bruyère, Filiot & Raskin

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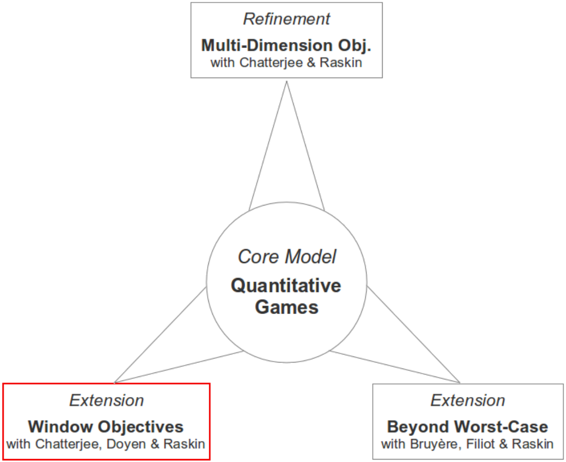
Window objectives



Mean-payoff and total-payoff have: **limited tractability** ($\in$ P ?? + multi TP undec.) and **no timing guarantee** (limit behavior)

Window objectives

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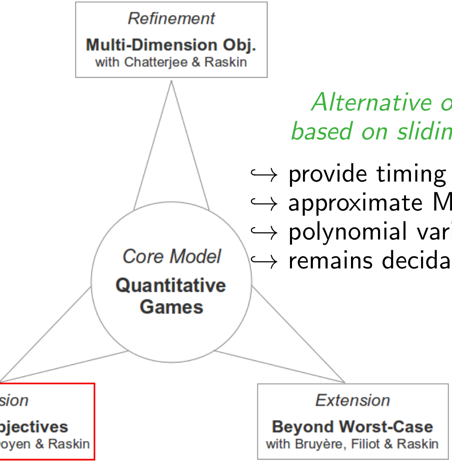

Alternative objectives based on sliding windows

- $\hookrightarrow$ provide timing guarantees
- $\hookrightarrow$ approximate MP and TP
- $\hookrightarrow$ polynomial variant in one-dim.
- $\hookrightarrow$ remains decidable in multi-dim.

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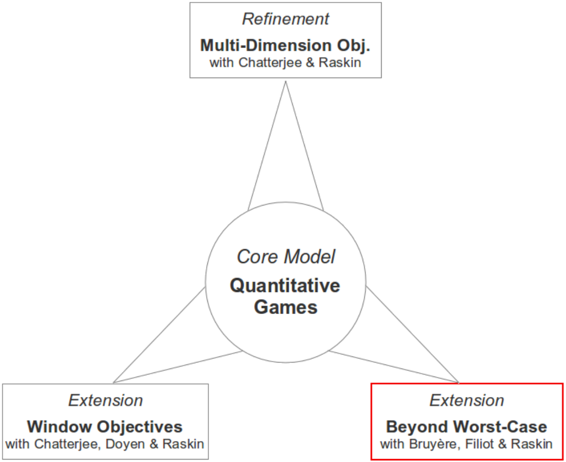
Beyond worst-case synthesis



Framework for the analysis of performance trade-offs w.r.t. the **nature of the environment**.

Beyond worst-case synthesis

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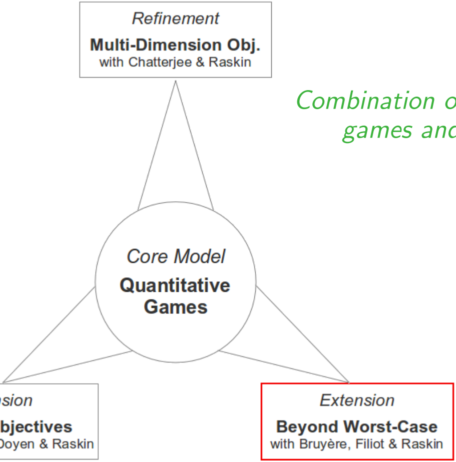


Combination of two views: games and MDPs

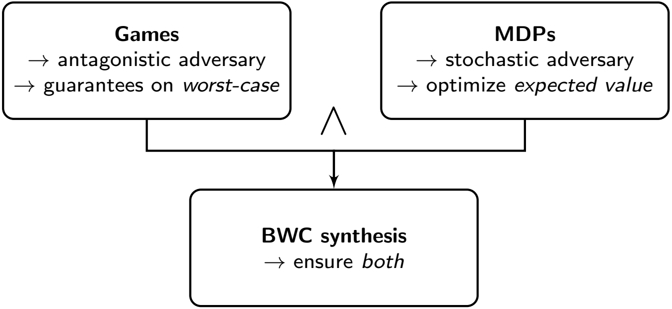
Framework for the analysis of performance trade-offs w.r.t. the **nature of the environment**.

Beyond worst-case synthesis

Games
→ antagonistic adversary
→ guarantees on *worst-case*

MDPs
→ stochastic adversary
→ optimize *expected value*

Beyond worst-case synthesis



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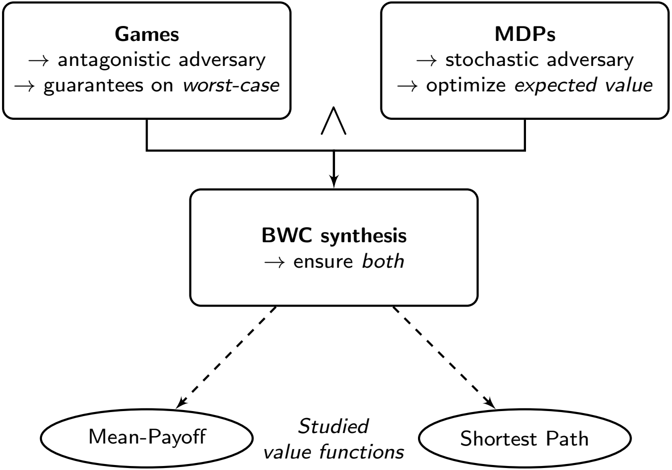
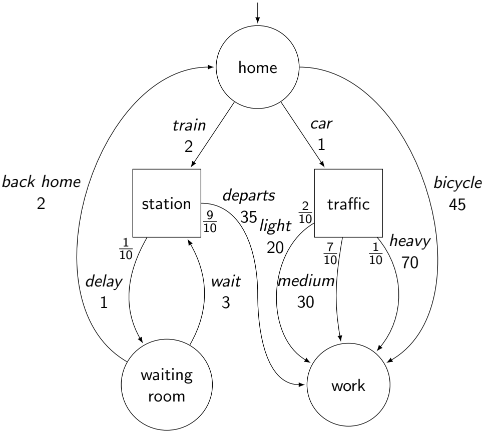


Example: going to work (shortest path)



- Weights = minutes
- Goal: minimize our **expected time** to reach "work"
- **But**, important meeting in one hour! Requires **strict guarantees on the worst-case** reaching time.

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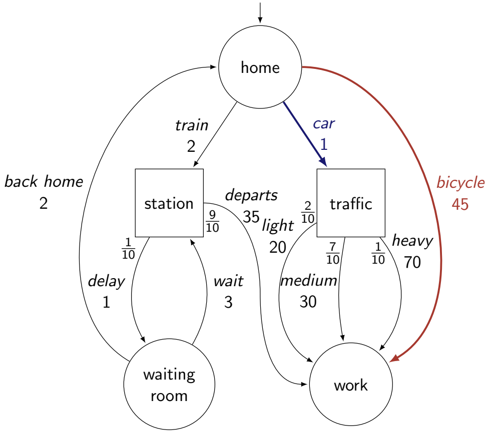

- Optimal **expectation** strategy: take the car.
  - $\mathbb{E} = 33$, $\text{WC} = 71 > 60$.
- Optimal **worst-case** strategy: bicycle.
  - $\mathbb{E} = \text{WC} = 45 < 60$.

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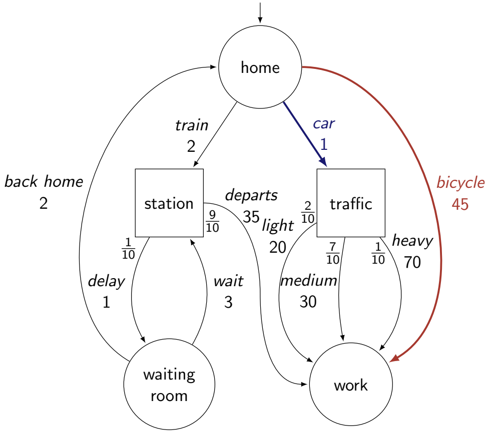

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- ▷ Optimal **worst-case** strategy: bicycle.
  - $\mathbb{E} = \text{WC} = 45 < 60$.
- ▷ **Sample BWC strategy**: try train up to 3 delays then switch to bicycle.
  - $\mathbb{E} \approx 37.45$, $\text{WC} = 58 < 60$.
  - Optimal $\mathbb{E}$ under WC constraint
  - Uses finite **memory**

BWC synthesis: overview

- **Mean-payoff**

	worst-case	expected value	**BWC**
complexity	$\mathsf{NP} \cap \mathsf{coNP}$	P-c.	**$\mathsf{NP} \cap \mathsf{coNP}$**
memory	pure memoryless		**pure pseudo-poly.**

▷ Additional modeling power **for free**!

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- **Shortest path**

	worst-case	expected value	**BWC**
complexity	P-c.		pseudo-poly./NP-hard
memory	pure memoryless		**pure pseudo-poly.**

▷ Problem **inherently harder** than worst-case and expectation.

Conclusion and future work

Key idea

We need innovative models to encompass the complexity of practical applications: trade-offs, strategies with rich guarantees...

Some challenges:

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- *Mixed objectives.*
- *Work toward a unifying meta-framework.*
  - ↪ Seems difficult in full generality but still room to extract common underlying principles to instantiate in specific settings.

Thank you!

Any question?

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Multi-dimension games

<table>
<tr><th></th><th></th><th>EG</th><th>$\underline{\mathsf{MP}}$</th><th>$\overline{\mathsf{MP}}$</th><th>$\underline{\mathsf{TP}}$</th><th>$\overline{\mathsf{TP}}$</th></tr>
<tr><td rowspan="3">one-dim.</td><td>complexity</td><td colspan="5">$\mathsf{NP} \cap \mathsf{coNP}$</td></tr>
<tr><td>$\mathcal{P}_1$ mem.</td><td colspan="5" rowspan="2">pure memoryless</td></tr>
<tr><td>$\mathcal{P}_2$ mem.</td></tr>
<tr><td rowspan="3">k-dim.</td><td>complexity</td><td colspan="2">coNP-c.</td><td>$\mathsf{NP} \cap \mathsf{coNP}$</td><td colspan="2">undec.</td></tr>
<tr><td>$\mathcal{P}_1$ mem.</td><td>pure finite</td><td colspan="2">pure infinite</td><td colspan="2" rowspan="2">-</td></tr>
<tr><td>$\mathcal{P}_2$ mem.</td><td colspan="3">pure memoryless</td></tr>
</table>

Randomness instead of memory?

	Multi EG and EG parity	Multi MP (parity)	MP parity
one-player	$\times$	$\surd$	$\surd$
two-player	$\times$	$\times$	$\surd$

Window objectives

	one-dimension			k-dimension		
	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.	complexity	$\mathcal{P}_1$ mem.	$\mathcal{P}_2$ mem.
$\underline{\text{MP}}$ / $\overline{\text{MP}}$	NP $\cap$ coNP	memoryless		coNP-c. / NP $\cap$ coNP	infinite	memoryless
$\underline{\text{TP}}$ / $\overline{\text{TP}}$	NP $\cap$ coNP	memoryless		**undec.**	-	-
WMP: fixed polynomial window	**P-c.**	**mem. req.** $\leq$ **linear(**$\lvert S \rvert \cdot l_{\max}$**)**		**PSPACE-h.** **EXP-easy**	**exponential**	
WMP: fixed arbitrary window	**P(**$\lvert S \rvert, V, l_{\max}$**)**			**EXP-c.**		
WMP: bounded window problem	**NP $\cap$ coNP**	**memoryless**	**infinite**	**NPR-h.**	-	-