Planning a Journey in an Uncertain Environment: Variations on the Stochastic Shortest Path Problem

Mickael Randour
LSV - CNRS & ENS Cachan / Complexys, UMONS

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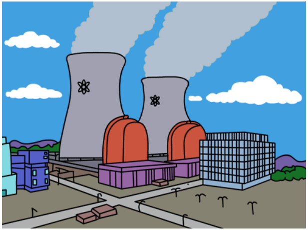
Controller synthesis



- Setting:
  - a reactive **system** to *control*,
  - an *interacting* **environment**,
  - a **specification** to *enforce*.

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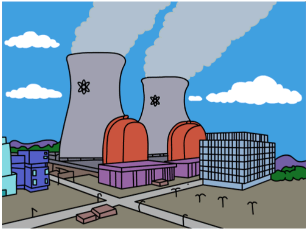

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  - ⇒ Need **formal methods**.

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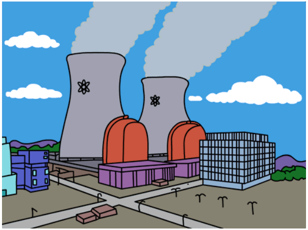

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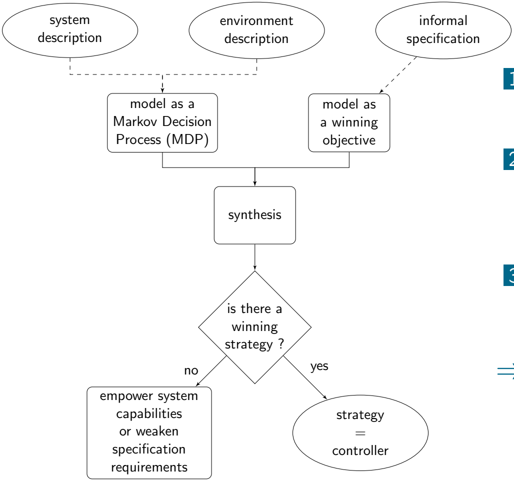
- For **critical** systems (e.g., airplane controller, power plants, ABS), testing is not enough!
  - ⇒ Need **formal methods**.
- **Automated synthesis** of provably-correct and efficient controllers:
  - ▷ mathematical frameworks,
    - ↪ e.g., game theory [GTW02, Ran13, Ran14]
  - ▷ software tools.

Strategy synthesis in stochastic environments

Strategy = formal model of how to control the system



1. How complex is it to decide if a winning strategy exists?
2. How complex such a strategy needs to be? **Simpler is better**.
3. Can we synthesize one efficiently?

⇒ Depends on the winning objective, the exact type of interaction, etc.

Aim of this talk

Flavor of $\neq$ types of **useful strategies** in stochastic environments.

- ▷ Joint paper[1] with J.-F. Raskin and O. Sankur (ULB) [RRS15b]
- ▷ Full paper available on arXiv: abs/1411.0835

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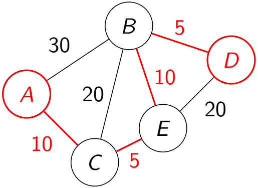
Applications to the **shortest path problem**.



↪ Find a **path of minimal length** in a weighted graph (Dijkstra, Bellman-Ford, etc) [CGR96].

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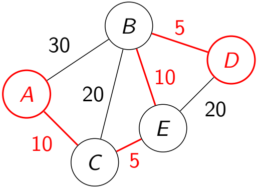

What if the environment is **uncertain**? E.g., in case of heavy traffic, some roads may be crowded.

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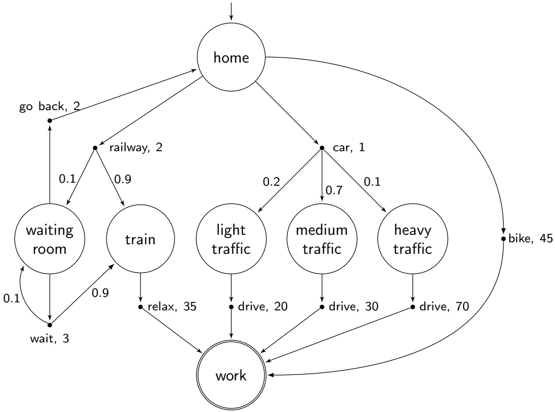
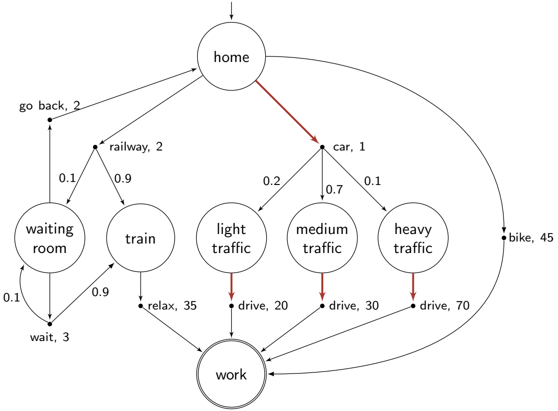
Planning a journey in an uncertain environment



Each action takes time, target = work.

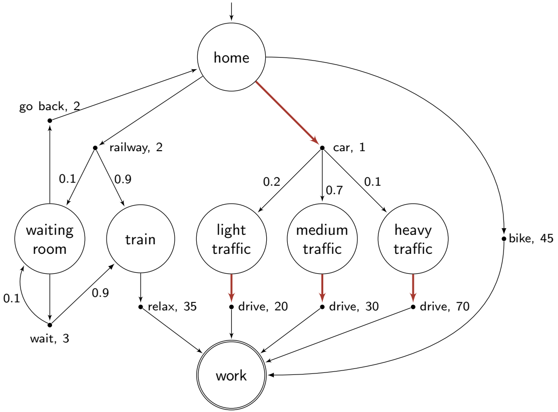
- ▷ What kind of strategies are we looking for when the environment is **stochastic** (MDP)?

Solution 1: minimize the *expected* time to work



- ▷ "Average" performance: meaningful when you journey often.
- ▷ **Simple strategies** suffice: no memory, no randomness.
- ▷ Taking the **car** is optimal: $\mathbb{E}_D^\sigma(\mathsf{TS}^{\mathsf{work}}) = 33$.

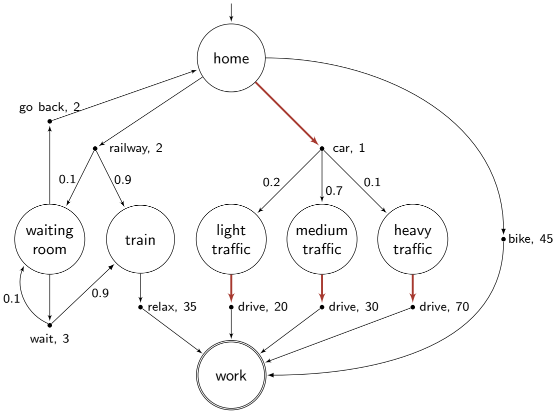
Solution 2: traveling without taking too many risks



Minimizing the *expected time* to destination makes sense **if** we travel often and **it is not a problem to be late**.

With car, in 10% of the cases, the journey takes 71 minutes.

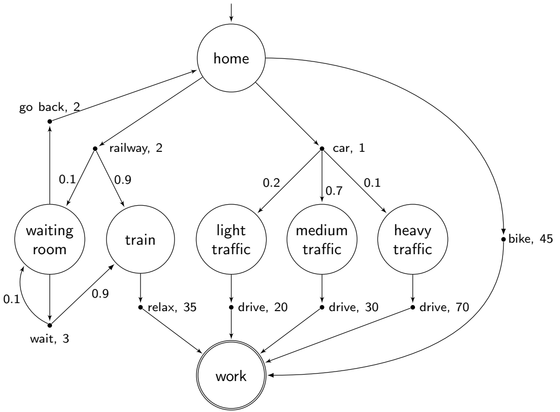
Solution 2: traveling without taking too many risks



Most bosses will not be happy if we are late too often. . .

⇝ what if we are risk-averse and want to avoid that?

Solution 2: maximize the *probability* to be on time



Specification: reach work within 40 minutes with 0.95 probability

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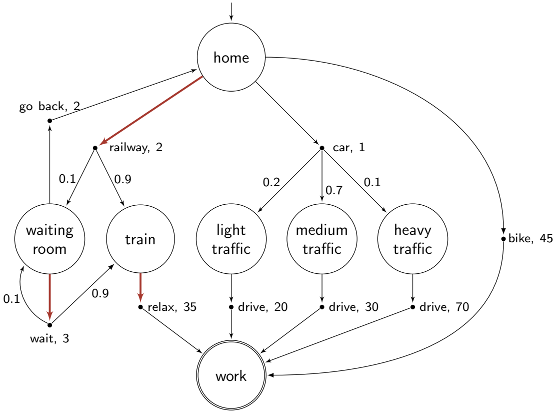
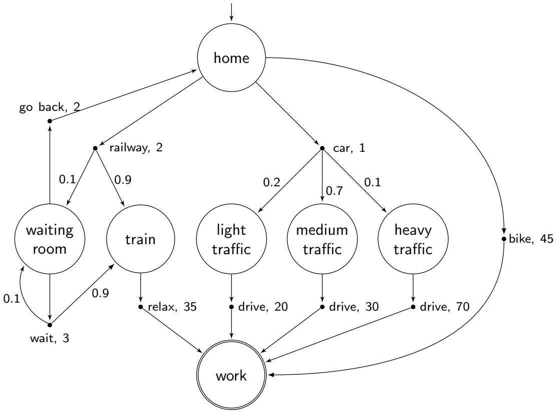


Specification: reach work within 40 minutes with 0.95 probability

Sample strategy: take the **train** $\rightsquigarrow \mathbb{P}^{\sigma}_{D}\left[\mathsf{TS}^{\mathsf{work}} \leq 40\right] = 0.99$

Bad choices: car (0.9) and bike (0.0)

Solution 3: strict worst-case guarantees



Specification: *guarantee* that work is reached within 60 minutes (to avoid missing an important meeting)

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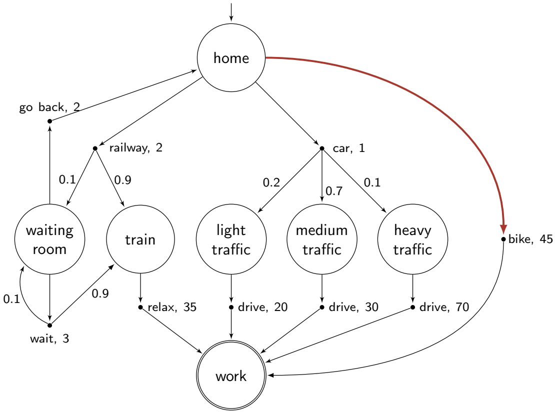
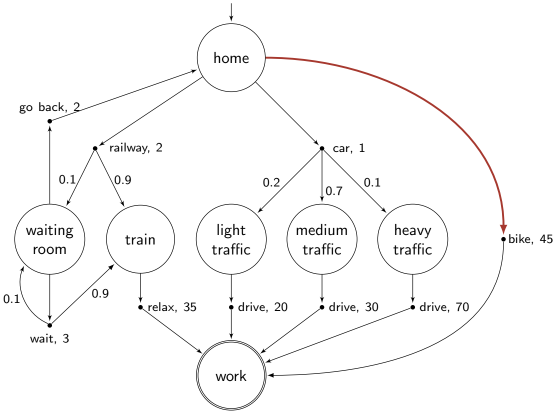


Specification: *guarantee* that work is reached within 60 minutes (to avoid missing an important meeting)

Sample strategy: **bike** $\leadsto$ worst-case reaching time $= 45$ minutes.
Bad choices: train ($wc = \infty$) and car ($wc = 71$)

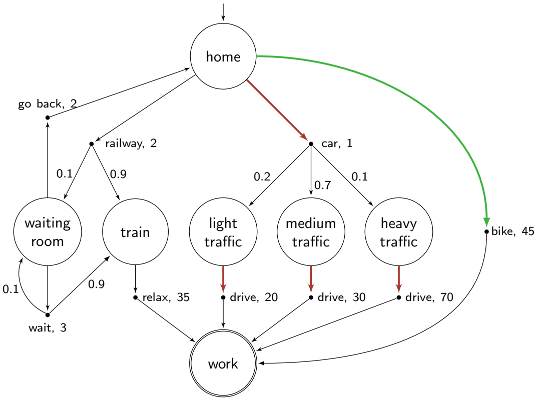
Solution 3: strict worst-case guarantees



Worst-case analysis $\rightsquigarrow$ **two-player game** against an antagonistic adversary (*bad guy*)

- ▷ forget about probabilities and give the choice of transitions to the adversary

Solution 4: minimize the *expected* time under strict worst-case guarantees



- Expected time: car $\rightsquigarrow \mathbb{E} = 33$ but *wc* $= 71 > 60$
- Worst-case: bike $\rightsquigarrow$ *wc* $= 45 < 60$ but $\mathbb{E} = 45 >>> 33$

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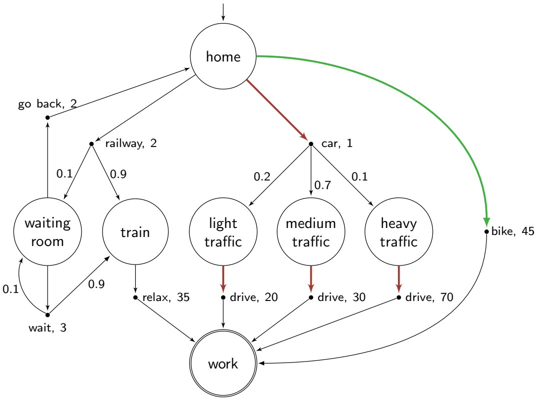

In practice, we want both! Can we do better?

▷ **Beyond worst-case synthesis** [BFRR14b, BFRR14a]: minimize the expected time under the worst-case constraint.

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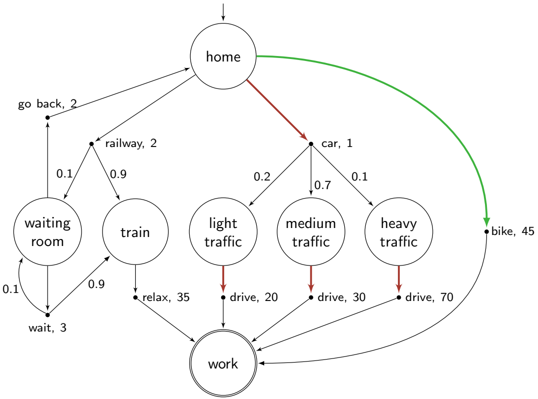
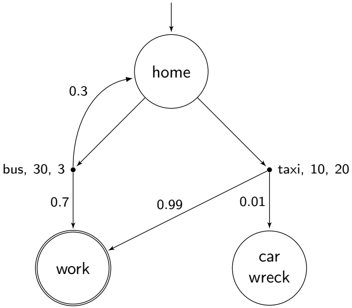


Sample strategy: try train up to 3 delays then switch to bike.

- $\rightsquigarrow$ $wc = 58 < 60$ and $\mathbb{E} \approx 37.34 << 45$
- $\rightsquigarrow$ Strategies need **memory** $\rightsquigarrow$ more complex!

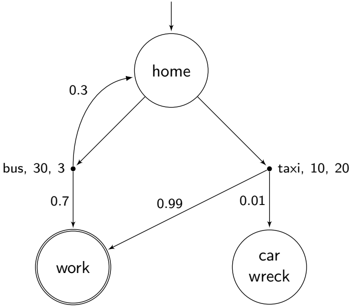
Solution 5: multiple objectives $\Rightarrow$ trade-offs



Two-dimensional weights on actions: *time* and *cost*.

Often necessary to consider trade-offs: e.g., between the probability to reach work in due time and the risks of an expensive journey.

Solution 5: multiple objectives ⇒ trade-offs



Solution 2 (probability) can only ensure a **single constraint**.

- **C1**: 80% of runs reach work in at most 40 minutes.
  - ▷ Taxi $\rightsquigarrow \leq 10$ minutes with probability $0.99 > 0.8$.

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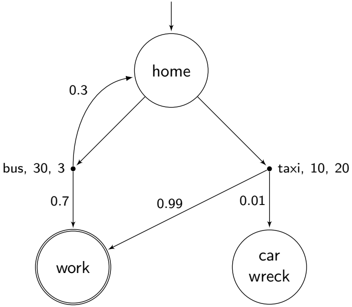

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- **C2**: 50% of them cost at most 10$ to reach work.
  - ▷ Bus $\rightsquigarrow \geq 70\%$ of the runs reach work for 3$.

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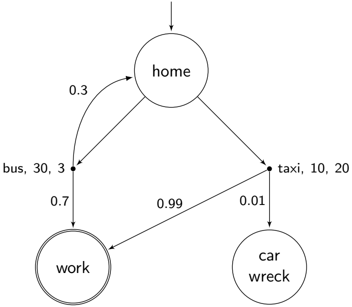

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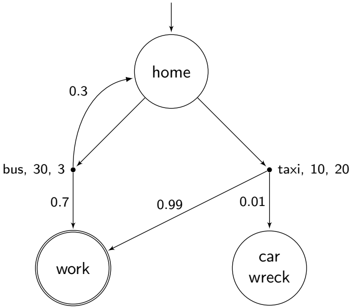
Taxi $\not\models$ C2, bus $\not\models$ C1. What if we want C1 $\wedge$ C2?

Solution 5: multiple objectives ⇒ trade-offs



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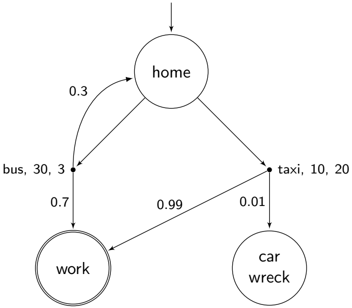
Study of **multi-constraint percentile queries** [RRS15a].

- ▷ Sample strategy: bus once, then taxi. Requires *memory*.
- ▷ Another strategy: bus with probability 3/5, taxi with probability 2/5. Requires *randomness*.

Solution 5: multiple objectives $\Rightarrow$ trade-offs



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Study of **multi-constraint percentile queries** [RRS15a].

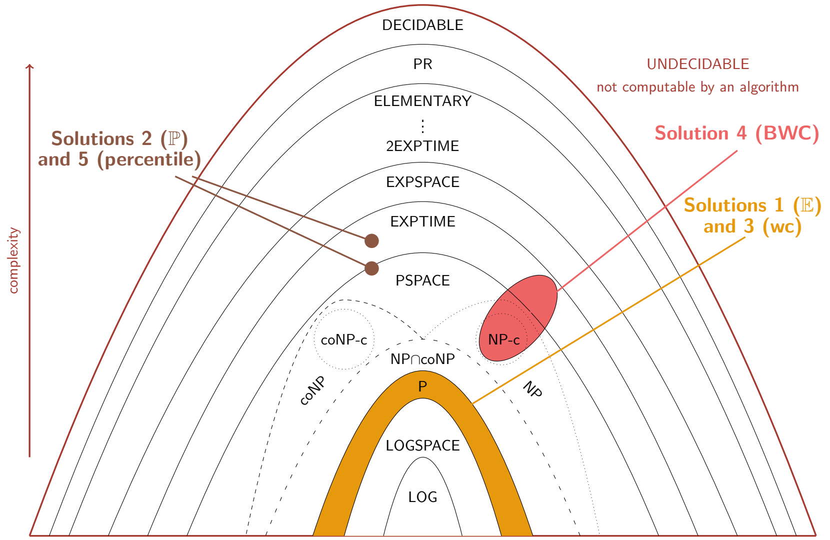
In general, *both* memory *and* randomness are required.

$\neq$ previous problems $\rightsquigarrow$ more complex!

Our research aims at:

- defining meaningful *strategy concepts*,
- providing *algorithms* and *tools* to compute those strategies,
- classifying the *complexity* of the different problems from a theoretical standpoint.
  - $\hookrightarrow$ Is it mathematically possible to obtain efficient algorithms?

Algorithmic complexity: hierarchy of problems



Thank you! Any question?

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Overview of theoretical results

SSP	complexity	strategy
SSP-E	PTIME	pure memoryless
SSP-P	pseudo-PTIME / PSPACE-h.	pure pseudo-poly.
SSP-G	PTIME	pure memoryless
SSP-WE	pseudo-PTIME / NP-h.	pure pseudo-poly.
SSP-PQ	EXPTIME (p.-PTIME) / PSPACE-h.	randomized exponential