Meet Your Expectations With Guarantees: Beyond Worst-Case Synthesis in Quantitative Games

V. Bruyère (UMONS) E. Filiot (ULB)
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Highlights of Logic, Games and Automata

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Context

- Verification and synthesis:
  - a reactive **system** to *control*,
  - an *interacting* **environment**,
  - a **specification** to *enforce*.

- Focus on *quantitative properties*.

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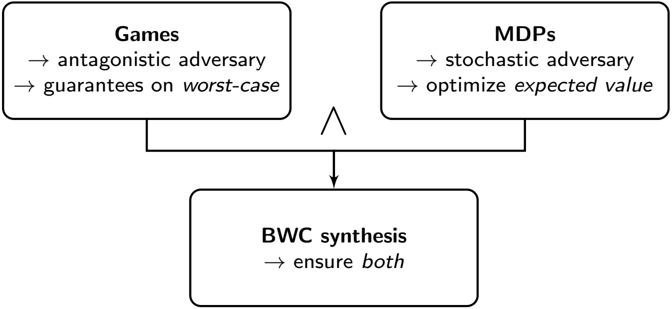
- Verification and synthesis:
  - a reactive **system** to *control*,
  - an *interacting* **environment**,
  - a **specification** to *enforce*.
- Focus on *quantitative properties*.
- Several ways to look at the interactions, and in particular, *the nature of the environment*.

Beyond worst-case synthesis

Games
$\rightarrow$ antagonistic adversary
$\rightarrow$ guarantees on *worst-case*

MDPs
$\rightarrow$ stochastic adversary
$\rightarrow$ optimize *expected value*

Beyond worst-case synthesis



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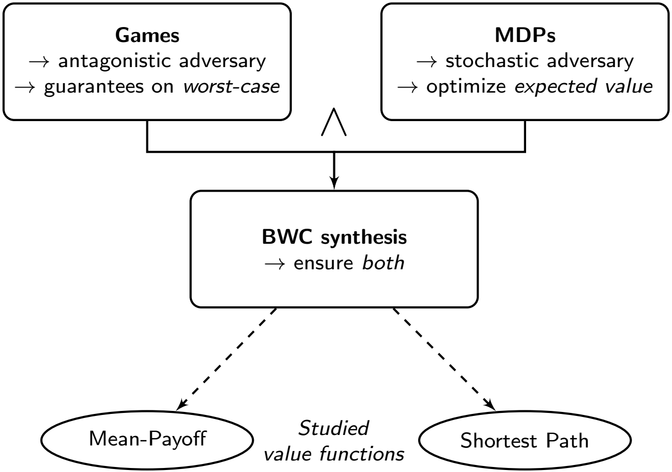

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Featured in STACS'14 [BFRR14]
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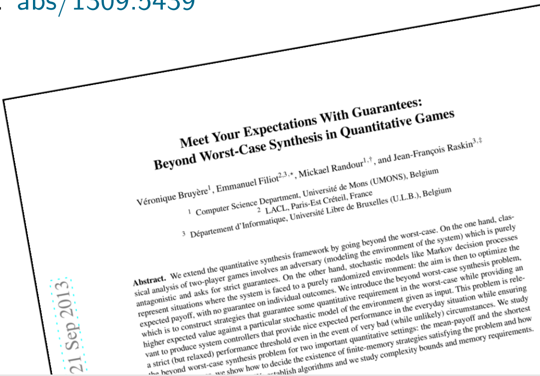

Meet Your Expectations With Guarantees:
Beyond Worst-Case Synthesis in Quantitative Games

Véronique Bruyère[1], Emmanuel Filiot[2,3,⋆], Mickael Randour[1,†], and Jean-François Raskin[3,‡]

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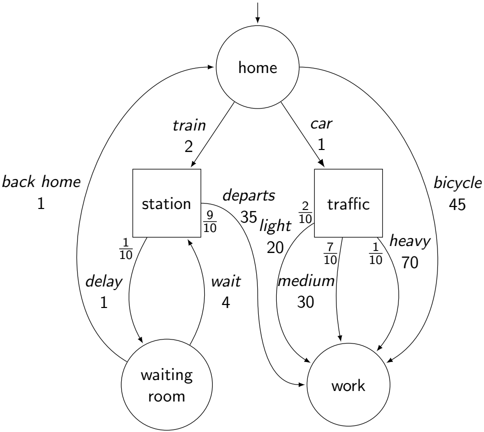
[2] LACL, Paris-Est Créteil, France
[3] Département d'Informatique, Université Libre de Bruxelles (U.L.B.), Belgium

Abstract. We extend the quantitative synthesis framework by going beyond the worst-case. On the one hand, classical analysis of two-player games involves an adversary (modeling the environment of the system) which is purely antagonistic and asks for strict guarantees. On the other hand, stochastic models like Markov decision processes represent situations where the system is faced to a purely randomized environment: the aim is then to optimize the expected payoff, with no guarantee on individual outcomes. We introduce the beyond worst-case synthesis problem, which is to construct strategies that guarantee some quantitative requirement in the worst-case while providing an higher expected value against a particular stochastic model of the environment given as input. This problem is relevant to produce system controllers that provide nice expected performance in the everyday situation while ensuring a strict (but relaxed) performance threshold even in the event of very bad (while unlikely) circumstances. We study the beyond worst-case synthesis problem for two important quantitative settings: the mean-payoff and the shortest … we show how to decide the existence of finite-memory strategies satisfying the problem and how … establish algorithms and we study complexity bounds and memory requirements.

Example: going to work (shortest path)



- Weights = minutes
- Goal: minimize our **expected time** to reach “work”
- **But**, important meeting in one hour! Requires **strict guarantees on the worst-case** reaching time.

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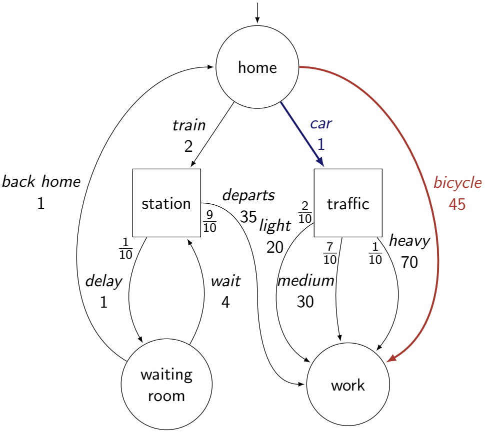

- Optimal **expectation** strategy: take the car.
  - $\mathbb{E} = 33$, $\mathrm{WC} = 71 > 60$.
- Optimal **worst-case** strategy: bicycle.
  - $\mathbb{E} = \mathrm{WC} = 45 < 60$.

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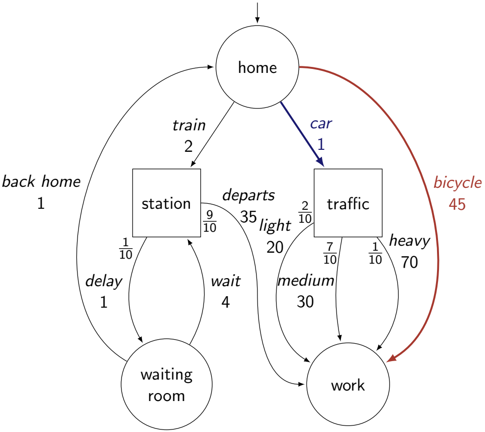

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- Optimal **worst-case** strategy: bicycle.
  - $\mathbb{E} = \text{WC} = 45 < 60$.
- **Sample BWC strategy**: try train up to 3 delays then switch to bicycle.
  - $\mathbb{E} \approx 37.56$, $\text{WC} = 59 < 60$.
  - Optimal $\mathbb{E}$ under WC constraint
  - Uses finite **memory**

Beyond worst-case synthesis

Definition

Given a game $G = (S_1, S_2, E, w)$, an initial state $s_{\text{init}} \in S$, a finite-memory stochastic model $\lambda_2^{\text{stoch}} \in \Lambda_2^F$ of the adversary, a measurable value function $f\colon \text{Plays}(G) \to \mathbb{R} \cup \{-\infty, \infty\}$, and two rational thresholds $\mu, \nu \in \mathbb{Q}$, the *beyond worst-case (BWC) problem* asks to decide if $\mathcal{P}_1$ has a finite-memory strategy $\lambda_1 \in \Lambda_1^F$ such that

$$\begin{cases} \forall\, \lambda_2 \in \Lambda_2,\ \forall\, \pi \in \text{Outs}_G(s_{\text{init}}, \lambda_1, \lambda_2),\ f(\pi) > \mu & (1) \\ \mathbb{E}_{s_{\text{init}}}^{G[\lambda_1, \lambda_2^{\text{stoch}}]}(f) > \nu & (2) \end{cases}$$

and the *BWC synthesis problem* asks to synthesize such a strategy if one exists.

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Notice the highlighted parts!

Related work

Common philosophy: avoiding outlier outcomes

1. Our strategies are *strongly risk averse*
   - ▷ avoid risk at all costs and optimize among safe strategies

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2. Other notions of risk ensure low probability of risked behavior [WL99, FKR95]
   - ▷ without worst-case guarantee
   - ▷ without good expectation

Related work

Common philosophy: avoiding outlier outcomes

1. Our strategies are *strongly risk averse*
   - ▷ avoid risk at all costs and optimize among safe strategies
2. Other notions of risk ensure low probability of risked behavior [WL99, FKR95]
   - ▷ without worst-case guarantee
   - ▷ without good expectation
3. Trade-off between expectation and variance [BCFK13, MT11]
   - ▷ statistical measure of the stability of the performance
   - ▷ no strict guarantee on individual outcomes

Mean-payoff

- $\mathsf{MP}(\pi) = \liminf_{n\to\infty} \left[\frac{1}{n} \cdot \sum_{i=0}^{i=n-1} w\big((s_i, s_{i+1})\big) \right]$

- Sample play $\pi = 2,\ -1,\ -4,\ 5,\ (2,\ 2,\ 5)^\omega$
  - ▷ $\mathsf{MP}(\pi) = 3$
  - ▷ long-run average weight $\rightsquigarrow$ *prefix-independent*

Mean-payoff

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	worst-case	expected value	**BWC**
complexity	NP ∩ coNP	P	**NP ∩ coNP**
memory	memoryless	memoryless	**pseudo-polynomial**

▷ [LL69, EM79, ZP96, Jur98, GS09, Put94, FV97]

▷ Additional modeling power **for free**!

Shortest path

- Strictly positive integer weights, $w\colon E \to \mathbb{N}_0$
- $\mathcal{P}_1$ wants to minimize its total cost up to target
  - ▷ inequalities are reversed

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	worst-case	expected value	**BWC**
complexity	P	P	**pseudo-poly. / NP-hard**
memory	memoryless	memoryless	**pseudo-poly.**

▷ [BT91, dA99]

▷ Problem **inherently harder** than worst-case and expectation.

▷ NP-hardness by K^{th} largest subset problem [JK78, GJ79]

Beyond BWC synthesis?

Possible future works include

- study of other quantitative objectives,
- extension of our results to more general settings (multi-dimension [CDHR10, CRR12], decidable classes of games with imperfect information [DDG+10], etc),
- other related strategies,
- application of the BWC problem to various practical cases.

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Thanks!

Do not hesitate to discuss with us!

References I

T. Brázdil, K. Chatterjee, V. Forejt, and A. Kucera.
Trading performance for stability in Markov decision processes.
In Proc. of LICS, pages 331–340. IEEE Computer Society, 2013.

V. Bruyère, E. Filiot, M. Randour, and J.-F. Raskin.
Meet your expectations with guarantees: beyond worst-case synthesis in quantitative games.
In Proc. of STACS, LIPIcs 25, pages 199–213. Schloss Dagstuhl - LZI, 2014.

D.P. Bertsekas and J.N. Tsitsiklis.
An analysis of stochastic shortest path problems.
Mathematics of Operations Research, 16:580–595, 1991.

K. Chatterjee, L. Doyen, T.A. Henzinger, and J.-F. Raskin.
Generalized mean-payoff and energy games.
In Proc. of FSTTCS, LIPIcs 8, pages 505–516. Schloss Dagstuhl - LZI, 2010.

K. Chatterjee, L. Doyen, M. Randour, and J.-F. Raskin.
Looking at mean-payoff and total-payoff through windows.
In Proc. of ATVA, LNCS 8172, pages 118–132. Springer, 2013.

K. Chatterjee, M. Randour, and J.-F. Raskin.
Strategy synthesis for multi-dimensional quantitative objectives.
In Proc. of CONCUR, LNCS 7454, pages 115–131. Springer, 2012.

L. de Alfaro.
Computing minimum and maximum reachability times in probabilistic systems.
In Proc. of CONCUR, LNCS 1664, pages 66–81. Springer, 1999.

References II

A. Degorre, L. Doyen, R. Gentilini, J.-F. Raskin, and S. Torunczyk.
Energy and mean-payoff games with imperfect information.
In Proc. of CSL, LNCS 6247, pages 260–274. Springer, 2010.

A. Ehrenfeucht and J. Mycielski.
Positional strategies for mean payoff games.
Int. Journal of Game Theory, 8(2):109–113, 1979.

J.A. Filar, D. Krass, and K.W. Ross.
Percentile performance criteria for limiting average Markov decision processes.
Transactions on Automatic Control, pages 2–10, 1995.

J. Filar and K. Vrieze.
Competitive Markov decision processes.
Springer, 1997.

M.R. Garey and D.S. Johnson.
Computers and intractability: a guide to the Theory of NP-Completeness.
Freeman New York, 1979.

P.W. Glynn and D. Ormoneit.
Hoeffding's inequality for uniformly ergodic Markov chains.
Statistics & Probability Letters, 56(2):143–146, 2002.

T. Gawlitza and H. Seidl.
Games through nested fixpoints.
In Proc. of CAV, LNCS 5643, pages 291–305. Springer, 2009.

References III

D.B. Johnson and S.D. Kashdan.
Lower bounds for selection in X + Y and other multisets.
Journal of the ACM, 25(4):556–570, 1978.

M. Jurdziński.
Deciding the winner in parity games is in UP ∩ co-UP.
Inf. Process. Lett., 68(3):119–124, 1998.

T.M. Liggett and S.A. Lippman.
Stochastic games with perfect information and time average payoff.
Siam Review, 11(4):604–607, 1969.

S. Mannor and J.N. Tsitsiklis.
Mean-variance optimization in Markov decision processes.
In Proc. of ICML, pages 177–184. Omnipress, 2011.

M.L. Puterman.
Markov decision processes: discrete stochastic dynamic programming.
John Wiley & Sons, Inc., New York, NY, USA, 1st edition, 1994.

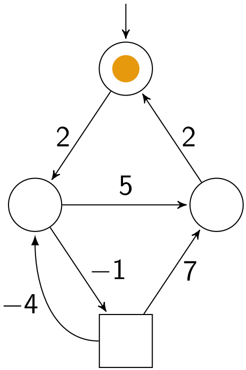
M. Tracol.
Fast convergence to state-action frequency polytopes for MDPs.
Oper. Res. Lett., 37(2):123–126, 2009.

C. Wu and Y. Lin.
Minimizing risk models in Markov decision processes with policies depending on target values.
Journal of Mathematical Analysis and Applications, 231(1):47–67, 1999.

References IV

U. Zwick and M. Paterson.
The complexity of mean payoff games on graphs.
Theoretical Computer Science, 158:343–359, 1996.

Quantitative games on graphs



- Graph $\mathcal{G} = (S, E, w)$ with $w \colon E \to \mathbb{Z}$
- Two-player *game* $G = (\mathcal{G}, S_1, S_2)$
  - ▷ $\mathcal{P}_1$ states = ◯
  - ▷ $\mathcal{P}_2$ states = □
- Plays have values
  - ▷ $f \colon \mathrm{Plays}(\mathcal{G}) \to \mathbb{R} \cup \{-\infty, \infty\}$
- Players follow *strategies*
  - ▷ $\lambda_i \colon \mathrm{Prefs}_i(G) \to \mathcal{D}(S)$
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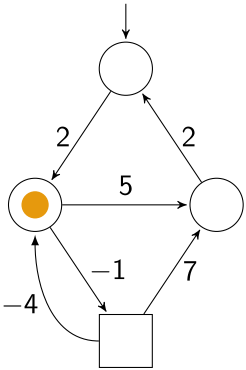

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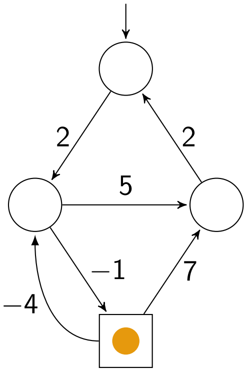

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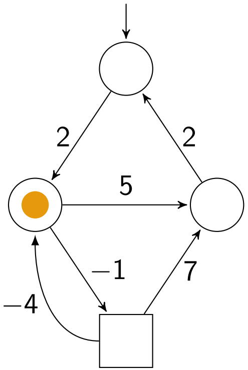

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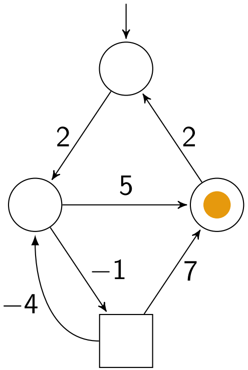

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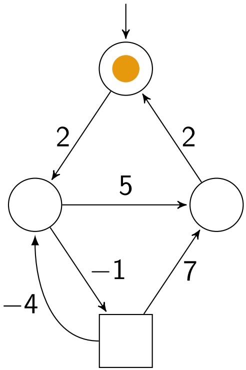

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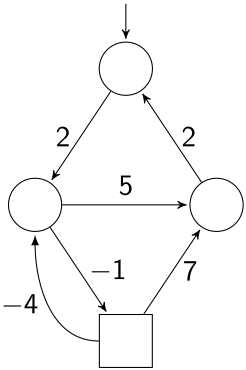

Then, $(2, 5, 2)^\omega$

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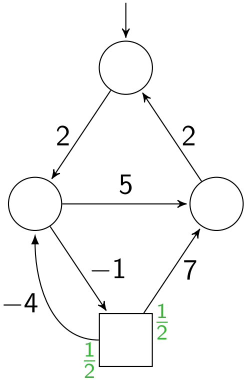
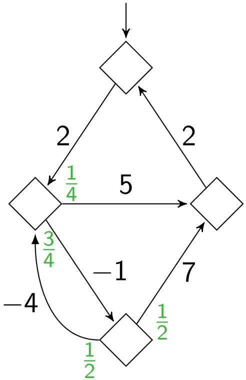
Markov decision processes



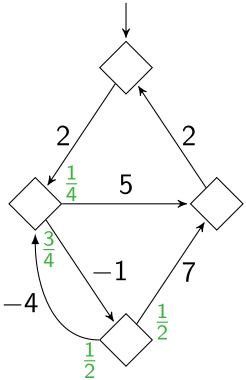
- MDP $P = (\mathcal{G}, S_1, S_\Delta, \Delta)$ with $\Delta \colon S_\Delta \to \mathcal{D}(S)$
  - $\mathcal{P}_1$ states $= \bigcirc$
  - stochastic states $= \square$
- MDP = game + strategy of $\mathcal{P}_2$
  - $P = G[\lambda_2]$

Markov chains



- MC $M = (\mathcal{G}, \delta)$ with $\delta\colon S \to \mathcal{D}(S)$
- MC $=$ MDP $+$ strategy of $\mathcal{P}_1$
  $=$ game $+$ both strategies
  - ▷ $M = P[\lambda_1] = G[\lambda_1, \lambda_2]$

Markov chains



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- MC = MDP + strategy of $\mathcal{P}_1$
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  - ▷ $M = P[\lambda_1] = G[\lambda_1, \lambda_2]$
- Event $\mathcal{A} \subseteq \text{Plays}(\mathcal{G})$
  - ▷ probability $\mathbb{P}^{M}_{s_{\text{init}}}(\mathcal{A})$
- Measurable $f \colon \text{Plays}(\mathcal{G}) \to \mathbb{R} \cup \{-\infty, \infty\}$
  - ▷ *expected value* $\mathbb{E}^{M}_{s_{\text{init}}}(f)$

Classical interpretations

- **System** trying to ensure a specification $= \mathcal{P}_1$
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  - ▷ *antagonistic*
    - two-player game, *worst-case* threshold problem for $\mu \in \mathbb{Q}$
    - $\exists ?\, \lambda_1 \in \Lambda_1,\ \forall\, \lambda_2 \in \Lambda_2,\ \forall\, \pi \in \mathsf{Outs}_G(s_{\mathsf{init}}, \lambda_1, \lambda_2),\ f(\pi) \geq \mu$

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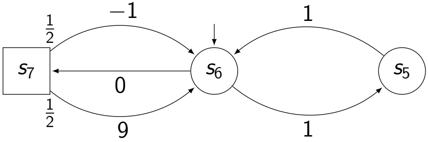
  - ▷ *fully stochastic*
    - MDP, *expected value* threshold problem for $\nu \in \mathbb{Q}$
    - $\exists ?\, \lambda_1 \in \Lambda_1,\, \mathbb{E}_{s_{\mathsf{init}}}^{P[\lambda_1]}(f) \geq \nu$

An ideal situation



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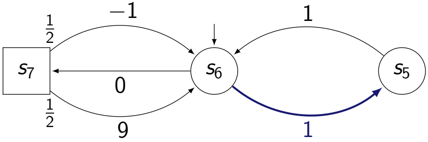
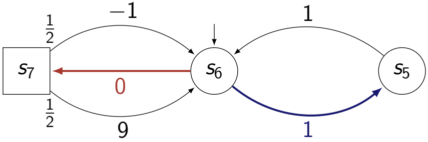


Game interpretation

- ▷ Worst-case threshold is $\mu = 0$
- ▷ **All** states are winning: memoryless optimal worst-case strategy $\lambda_1^{wc} \in \Lambda_1^{PM}(G)$, ensuring $\mu^* = 1 > 0$

An ideal situation



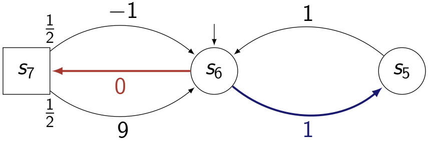
Game interpretation

- ▷ Worst-case threshold is $\mu = 0$
- ▷ **All** states are winning: memoryless optimal worst-case strategy $\lambda_1^{wc} \in \Lambda_1^{PM}(G)$, ensuring $\mu^* = 1 > 0$

MDP interpretation

- ▷ Memoryless optimal expected value strategy $\lambda_1^{e} \in \Lambda_1^{PM}(P)$ achieves $\nu^* = 2$

A cornerstone of our approach



BWC problem: **what kind of threholds $(0, \nu)$ can we achieve?**

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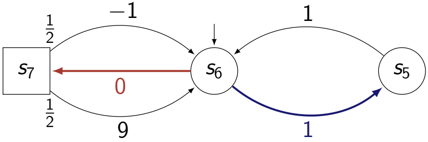


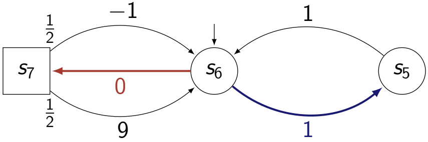
BWC problem: **what kind of threholds $(0, \nu)$ can we achieve?**

Key result

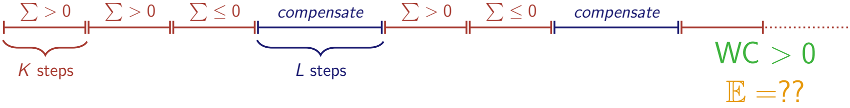
For all $\varepsilon > 0$, there exists a finite-memory strategy of $\mathcal{P}_1$ that satisfies the BWC problem for the thresholds pair $(0, \nu^* - \varepsilon)$.

▷ We can be **arbitrarily close to the optimal expectation** while ensuring the worst-case!

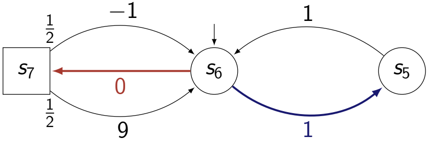
Combined strategy



Outcomes of the form



Combined strategy



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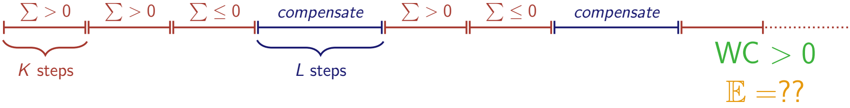
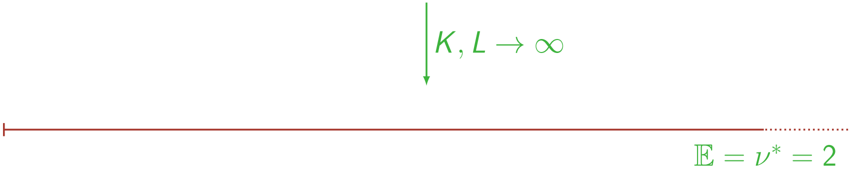


What we want



Combined strategy: crux of the proof

Precise reasoning on convergence rates using involved techniques

- When K grows, L needs to grow linearly to ensure WC

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  - ▷ application of Chernoff bounds and Hoeffding's inequality for Markov chains [Tra09, GO02]

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Precise reasoning on convergence rates using involved techniques

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- When K grows, $\mathbb{P}(\vdash\!\!\!\longrightarrow\!\!\dashv) \to 0$ and it decreases exponentially fast
  - ▷ application of Chernoff bounds and Hoeffding's inequality for Markov chains [Tra09, GO02]
- Overall we are good: WC > 0 and $\mathbb{E} > \nu^* - \varepsilon$ for sufficiently large K, L.