Strategy Synthesis for Multi-dimensional Quantitative Objectives

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Aim of this work

Controller synthesis

- functional properties
- quantitative requirements

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Controller synthesis

- ▷ functional properties
- ▷ quantitative requirements

Implementable controllers $\rightsquigarrow$ restriction to finite-memory strategies.

Aim of this work

- Study games with
  - ▷ multi-dimensional quantitative objectives (energy and mean-payoff)
  - ▷ *and* a parity objective.

$\rightsquigarrow$ First study of such a conjunction.

- Address questions that revolve around *strategies*:
  - ▷ bounds on memory,
  - ▷ synthesis algorithm,
  - ▷ randomness $\overset{?}{\sim}$ memory.

Results Overview

- Memory bounds

MEPGs	MMPPGs	
optimal	finite-memory optimal	optimal
exp.	**exp.**	infinite [CDHR10]

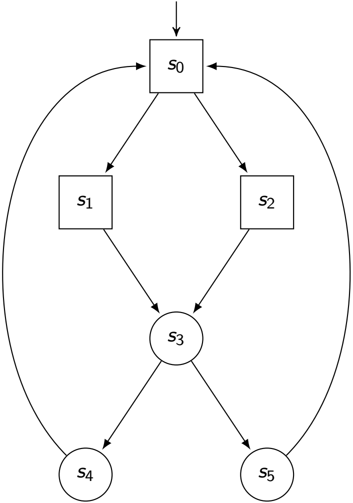
- Strategy synthesis (finite memory)

MEPGs	MMPPGs
EXPTIME	**EXPTIME**

- Randomness as a substitute for finite memory

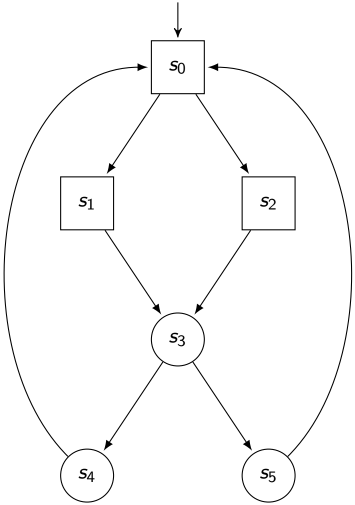
	MEGs	EPGs	MMP(P)Gs	MPPGs
one-player	×	×	√	√
two-player	×	×	×	√

Turn-based games



- $G = (S_1, S_2, s_{init}, E)$
  $S = S_1 \cup S_2, S_1 \cap S_2 = \emptyset, E \subseteq S \times S$
- $\mathcal{P}_1$ states = ◯
- $\mathcal{P}_2$ states = □

Turn-based games



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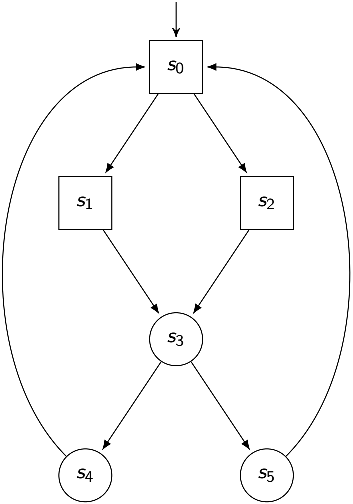
- $\mathcal{P}_1$ states = ○
- $\mathcal{P}_2$ states = □
- Play $\pi = s^0 s^1 s^2 \dots s^n \dots$ s.t. $s^0 = s_{init}$
- Prefix $\rho = \pi(n) = s^0 s^1 s^2 \dots s^n$

Pure strategies



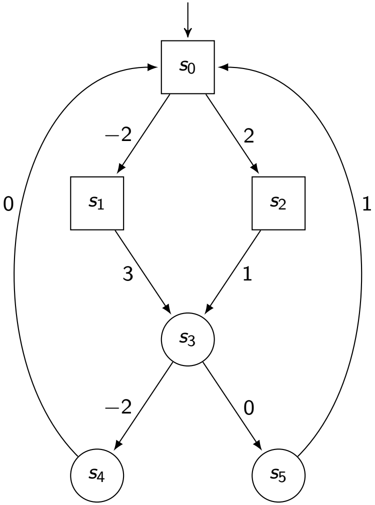
- Pure strategy for $\mathcal{P}_i$

  $\lambda_i \in \Lambda_i : \text{Prefs}_i(G) \rightarrow S$ s.t. for all $\rho \in \text{Prefs}_i(G)$, $(\text{Last}(\rho), \lambda_i(\rho)) \in E$
- Memoryless strategy

  $\lambda_i^{pm} \in \Lambda_i^{PM} : S_i \rightarrow S$
- Finite-memory strategy

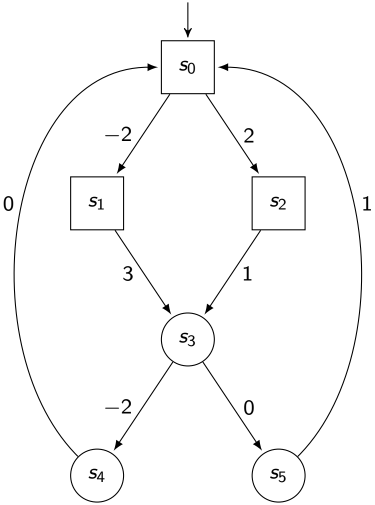
  $\lambda_i^{fm} \in \Lambda_i^{FM} : \text{Prefs}_i(G) \rightarrow S$, and can be encoded as a deterministic Moore machine

Integer payoff function



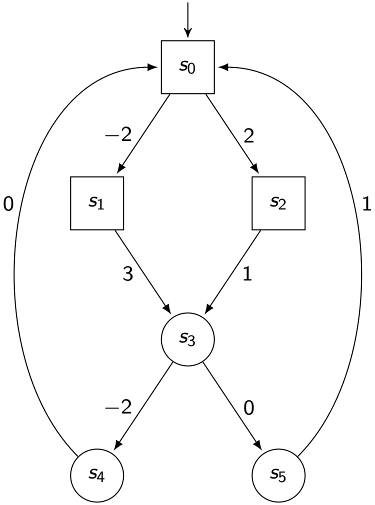
- $G = (S_1, S_2, s_{init}, E, \underline{w})$
- $w : E \to \mathbb{Z}$

Integer payoff function



- $G = (S_1, S_2, s_{init}, E, \underline{w})$
- $w : E \to \mathbb{Z}$
- *Energy level*
  $\mathrm{EL}(\rho) = \sum_{i=0}^{i=n-1} w(s_i, s_{i+1})$
- *Mean-payoff*
  $\mathrm{MP}(\pi) = \liminf_{n\to\infty} \frac{1}{n}\mathrm{EL}(\pi(n))$

Energy and mean-payoff objectives



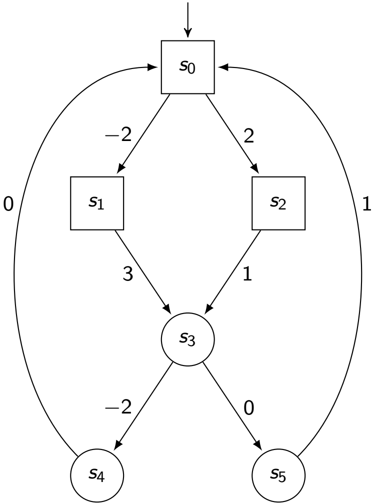
- **Energy objective**

  Given initial credit $v_0 \in \mathbb{N}$,
  $\mathsf{PosEnergy}_G(v_0) = \{\pi \in \mathsf{Plays}(G) \mid \forall\, n \geq 0 : v_0 + \mathsf{EL}(\pi(n)) \in \mathbb{N}\}$

- **Mean-payoff objective**

  Given threshold $v \in \mathbb{Q}$,
  $\mathsf{MeanPayoff}_G(v) = \{\pi \in \mathsf{Plays}(G) \mid \mathsf{MP}(\pi) \geq v\}$

Energy and mean-payoff objectives



- $\lambda_1(s_3) = s_4$
  - ▷ λ_1 wins for $\text{MeanPayoff}_G(\frac{-1}{4})$
  - ▷ λ_1 loses for $\text{PosEnergy}_G(v_0)$, for any arbitrary high initial credit
- $\lambda_1(s_3) = s_5$
  - ▷ λ_1 wins for $\text{MeanPayoff}_G(\frac{1}{2})$
  - ▷ λ_1 wins for $\text{PosEnergy}_G(v_0)$, with $v_0 = 2$

Decision problems

- *Unknown initial credit problem*:

  $\exists ?\, v_0 \in \mathbb{N}, \lambda_1 \in \Lambda_1$ s.t. λ_1 wins for $\mathsf{PosEnergy}_G(v_0)$
- *Mean-payoff threshold problem*:

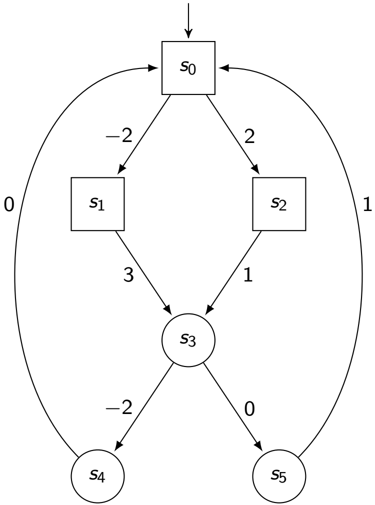
  Given $v \in \mathbb{Q}$, $\exists ?\, \lambda_1 \in \Lambda_1$ s.t. λ_1 wins for $\mathsf{MeanPayoff}_G(v)$

MPG threshold v problem equivalent to EG$-v$ unknown initial credit problem [BFL+08].

Complexity of EGs and MPGs

	EGs	MPGs
Memory to win	memoryless [CdAHS03, BFL+08]	memoryless [EM79, LL69]
Decision problem	NP ∩ coNP	NP ∩ coNP

Multi-dimensional weights



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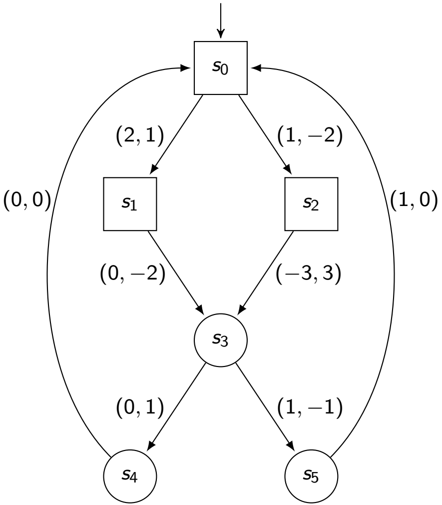
Multi-dimensional weights



- $G = (S_1, S_2, s_{init}, E, \underline{k}, w)$
- $w : E \to \mathbb{Z}^k$

▷ multiple quantitative aspects

▷ natural extensions of energy and mean-payoff objectives and associated decision problems

MEGs & MMPGs

- Finite memory suffice for MEGs [CDHR10].

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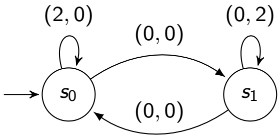
- However, infinite memory is needed for MMPGs, even with only one player! [CDHR10]



- ▷ To obtain $\mathrm{MP}(\pi) = (1, 1)$, $\mathcal{P}_1$ has to visit s_0 and s_1 for longer and longer intervals before jumping from one to the other.
- ▷ Any finite-memory strategy induces an ultimately periodic play s.t. $\mathrm{MP}(\pi) = (x, y)$, $x + y < 2$.
- ▷ With $\limsup$ as MP the gap is huge : $(2, 2)$.

MEGs & MMPGs

- If players are restricted to finite memory [CDHR10],
  - ▷ MEGs and MMPGs are still determined and they are log-space equivalent,
  - ▷ the unknown initial credit and the mean-payoff threshold problems are coNP-complete,
  - ▷ no clue on memory bounds for $\mathcal{P}_1$ (for $\mathcal{P}_2$, we know it is memoryless).

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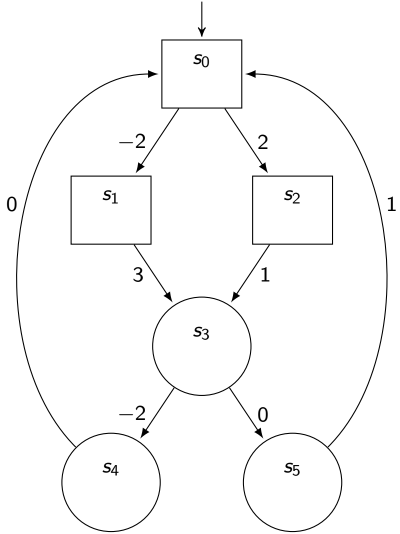
  - no clue on memory bounds for $\mathcal{P}_1$ (for $\mathcal{P}_2$, we know it is memoryless).
- Other interesting results on decision problems on MEGs are proved in [FJLS11]. Surprisingly, given a fixed initial vector, the problem becomes EXPSPACE-hard.

Parity



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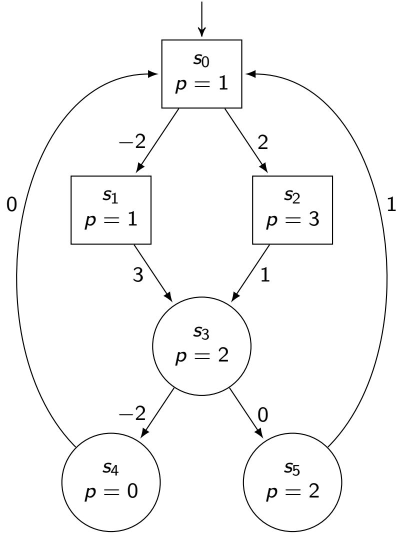
Parity



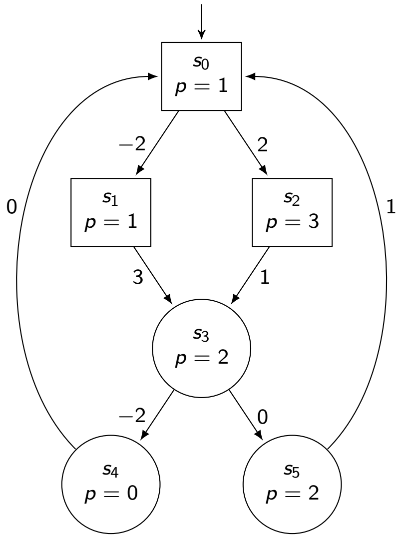
- $G_p = (S_1, S_2, s_{init}, E, w, \underline{p})$
- $p : S \to \mathbb{N}$
- $\text{Par}(\pi) = \min \{p(s) \mid s \in \text{Inf}(\pi)\}$

▷ $\text{Parity}_{G_p} = \{\pi \in \text{Plays}(G_p) \mid \text{Par}(\pi) \bmod 2 = 0\}$

▷ canonical way to express ω-regular objectives

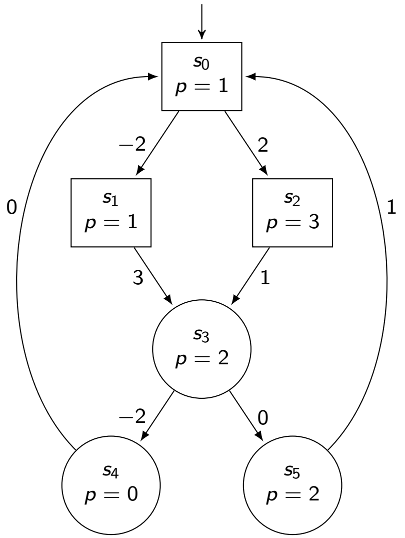
▷ achieve the energy or mean-payoff objective while satisfying the parity condition

Parity



- To win the energy parity objective, $\mathcal{P}_1$ must
  - visit s_4 infinitely often,
  - alternate with visits of s_5 to fund future visits of s_4.

Parity



- To win the energy parity objective, $\mathcal{P}_1$ must
  - visit s_4 infinitely often,
  - alternate with visits of s_5 to fund future visits of s_4.
- To achieve optimality for the mean-payoff parity objective, $\mathcal{P}_1$ should wait longer and longer between visits of s_4.

EPGs & MPPGs

- Exponential memory suffice for EPGs and deciding the winner is in NP $\cap$ coNP [CD10].

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EPGs & MPPGs

- Exponential memory suffice for EPGs and deciding the winner is in NP $\cap$ coNP [CD10].
- Infinite memory is needed for MPPGs and deciding the winner is in NP $\cap$ coNP [CHJ05, BMOU11].
- Finite-memory ε-optimal strategies for MPPGs always exist [BCHJ09].
- $\mathcal{P}_1$ has a winning strategy for the MPPG $\langle G, p, w \rangle$ iff $\mathcal{P}_1$ has a winning strategy for the EPG $\langle G, p, w + \varepsilon \rangle$, with $\varepsilon = \frac{1}{|S|+1}$ [CD10].

Restriction to finite memory

- Infinite memory:
  - ▷ needed for MMPGs & MPPGs,
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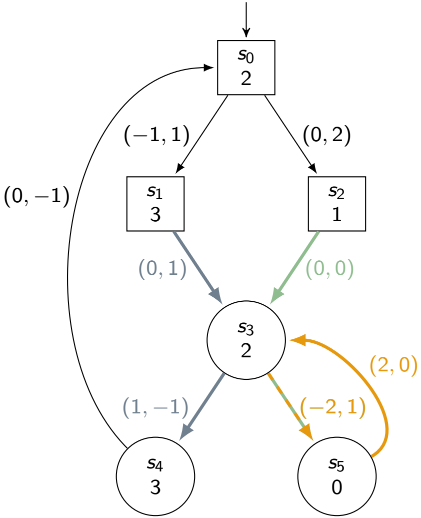
  - ▷ the way to go for strategy synthesis.
- **Our goals:**
  - ▷ bounds on memory,
  - ▷ strategy synthesis algorithm,
  - ▷ encoding of memory as randomness.

Obtained results

MEPGs	MMPPGs	
optimal	finite-memory optimal	optimal
exp.	**exp.**	infinite [CDHR10]

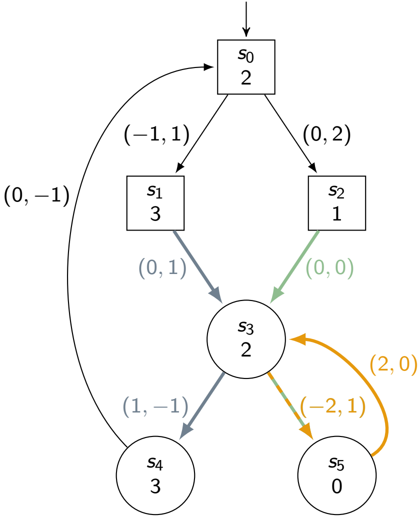
By [CDHR10], we only have to consider MEPGs. Recall that the unknown initial credit decision problem for MEGs (without parity) is coNP-complete.

Upper memory bound: even-parity SCTs



- A winning strategy λ_1 for initial credit $v_0 = (2, 0)$ is
  - ▷ $\lambda_1(*s_1 s_3) = s_4$,
  - ▷ $\lambda_1(*s_2 s_3) = s_5$,
  - ▷ $\lambda_1(*s_5 s_3) = s_5$.

Upper memory bound: even-parity SCTs



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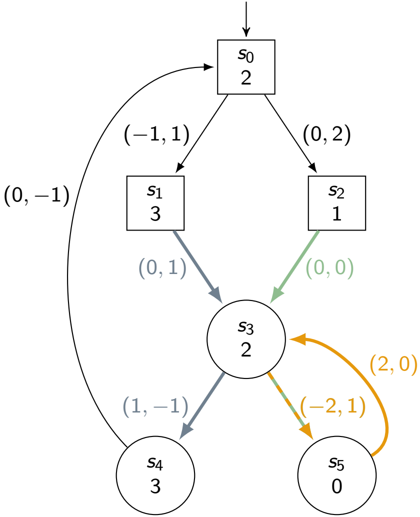
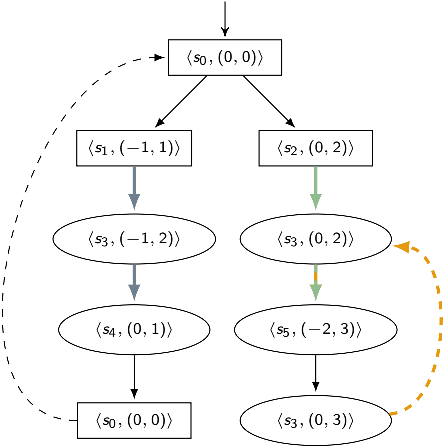
  - $\lambda_1(*s_2 s_3) = s_5$,
  - $\lambda_1(*s_5 s_3) = s_5$.
- Lemma: To win, $\mathcal{P}_1$ must be able to enforce positive cycles of even parity.
  - Self-covering paths on VASS [Rac78, RY86].
  - *Self-covering trees (SCTs)* on reachability games over VASS [BJK10].

Upper memory bound: even-parity SCTs



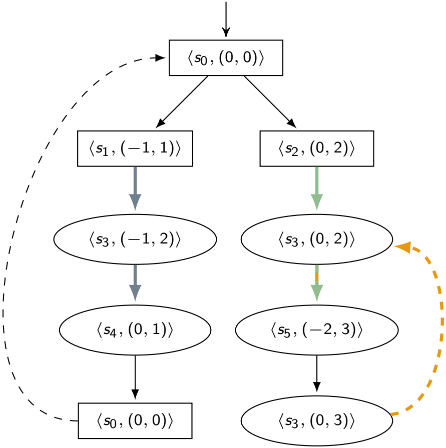


Pebble moves $\Rightarrow$ strategy.

Upper memory bound: even-parity SCTs

$T = (Q, R)$ is an epSCT for s_0, $\Theta : Q \mapsto S \times \mathbb{Z}^k$ is a labeling function.

- Root labeled $\langle s_0, (0, \ldots, 0)\rangle$.
- Non-leaf nodes have
  - ▷ unique child if $\mathcal{P}_1$,
  - ▷ all possible children if $\mathcal{P}_2$.
- Leafs have *even-descendance energy ancestors*: ancestors with lower label and minimal priority even on the downward path.



Pebble moves $\Rightarrow$ strategy.

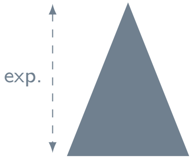
Upper memory bound: SCTs for VASS games

Theorem (application of [BJK10]): *On a VASS game with weights in* $\{-1, 0, 1\}^k$*, if state* s *is winning for* $\mathcal{P}_1$*, there is a SCT for* s *of* *depth* *at most* $l = 2^{(d-1)\cdot|S|} \cdot (|S| + 1)^{c\cdot k^2}$*, with* c *a constant independent of the considered VASS game and* d *its branching degree.*

⇝ If there exists a winning strategy, there exists a “compact” one.

⇝ Idea is to eliminate unnecessary cycles.

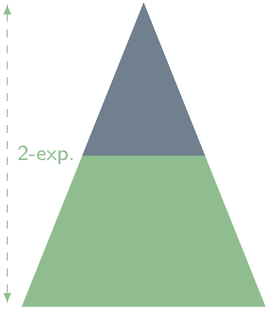
Upper memory bound: SCTs for MEGs (no parity)



$$w : E \rightarrow \{-1, 0, 1\}^k$$
$$l = 2^{(d-1)\cdot|S|} \cdot (|S| + 1)^{c\cdot k^2}$$

Depth bound from [BJK10].

Upper memory bound: SCTs for MEGs (no parity)



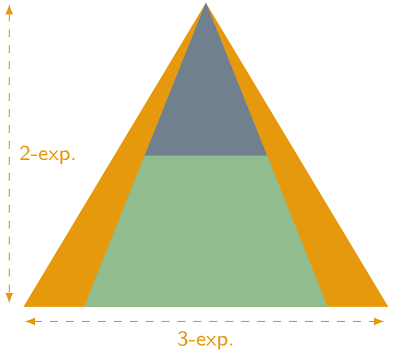
$$w : E \rightarrow \{-1, 0, 1\}^k$$
$$l = 2^{(d-1)\cdot|S|} \cdot (|S| + 1)^{c\cdot k^2}$$
$$\Downarrow$$

$w : E \rightarrow \mathbb{Z}^k$, W max absolute weight,
V bits to encode W

$$l = 2^{(d-1)\cdot W\cdot|S|} \cdot (W \cdot |S| + 1)^{c\cdot k^2}$$
$$= 2^{(d-1)\cdot 2^V\cdot|S|} \cdot (W \cdot |S| + 1)^{c\cdot k^2}$$

Naive approach: blow-up by W in the size of the state space.

Upper memory bound: SCTs for MEGs (no parity)



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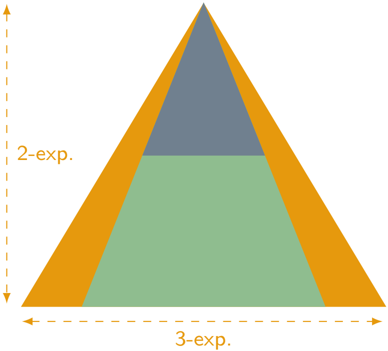
$$\Downarrow$$

Width bounded by $L = d^l$

Naive approach: width increases exponentially with depth.

Upper memory bound: SCTs for MEGs (no parity)



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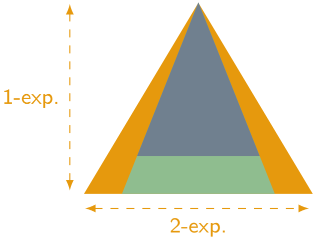
$$\Downarrow$$
Width bounded by $L = d^l$

Naive approach: overall, 3-exp. memory $\leq L \cdot l$, without parity.

Upper memory bound: epSCTs for MEPGs



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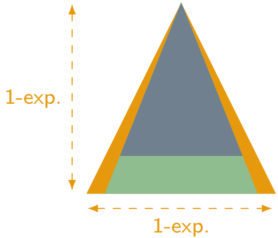
$$\Downarrow$$
Width bounded by $L = d^l$

Refined approach: no blow-up in exponent as branching is preserved, extension to parity.

Upper memory bound: epSCTs for MEPGs



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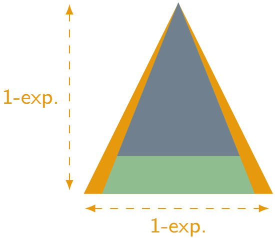
$$\Downarrow$$
Width bounded by $L = |S| \cdot (2 \cdot l \cdot W + 1)^k$

Refined approach: merge equivalent nodes, width is bounded by number of incomparable labels (see next slide).

Upper memory bound: epSCTs for MEPGs



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$$\Downarrow$$
Width bounded by $L = |S| \cdot (2 \cdot l \cdot W + 1)^k$

Refined approach: overall, **single exp. memory** $\leq L \cdot l$, for multi energy *along with* parity. Initial credit bounded by $l \cdot W$.

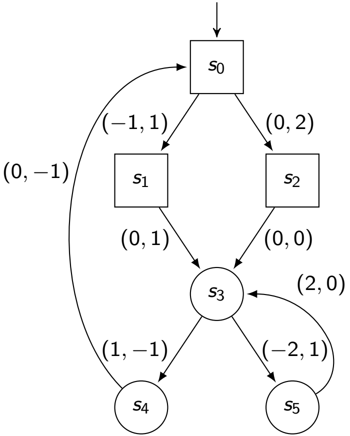
Upper memory bound: from MEPGs to MEGs

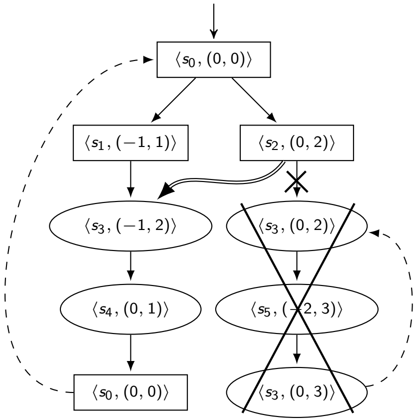
- Thanks to the bound on depth for MEPGs, encode parity ($2 \cdot m$ priorities) as m additional energy dimensions.
  - ▷ For each odd priority, add one dimension.
  - ▷ Decrease by 1 when this odd priority is visited.
  - ▷ Increase by l each time a smaller even priority is visited.

- $\mathcal{P}_1$ maintains the energy positive on all additional dimensions iff he wins the original parity objective.

Upper memory bound: merging nodes in SCTs

- Key idea to reduce width to single exp.
  - ▷ $\mathcal{P}_1$ only cares about the energy level.
  - ▷ If he can win with energy v, he can win with energy $\geq v$.

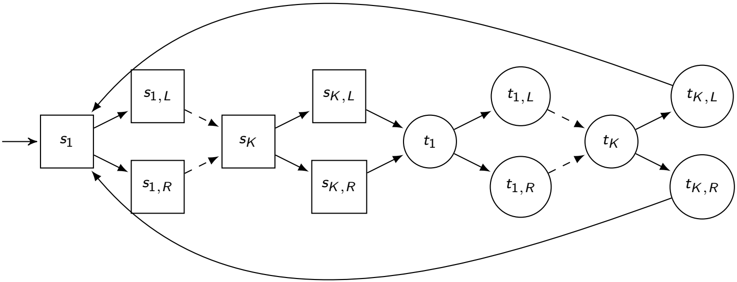




Lower memory bound

Lemma: *There exists a family of multi energy games* $(G(K))_{K\geq 1,} = (S_1, S_2, s_{init}, E, k = 2 \cdot K, w : E \to \{-1, 0, 1\})$ *s.t. for any initial credit,* $\mathcal{P}_1$ *needs exponential memory to win.*

Lower memory bound

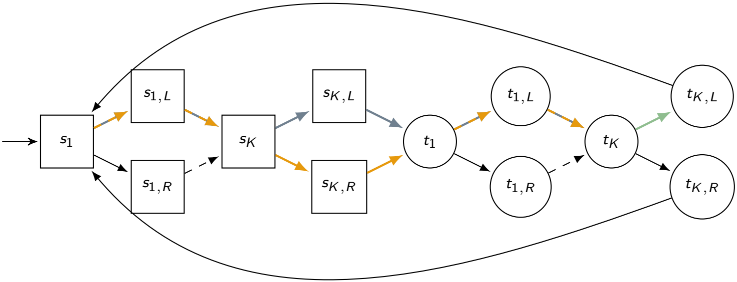


$$\forall\, 1 \leq i \leq K, w((\circ, s_i)) = w((\circ, t_i)) = (0, \ldots, 0),$$

$$w((s_i, s_{i,L})) = -w((s_i, s_{i,R})) = w((t_i, t_{i,L})) = -w((t_i, t_{i,R})),$$

$$\forall\, 1 \leq j \leq k,\ w((s_i, s_{i,L}))(j) = \begin{cases} = 1 \text{ if } j = 2 \cdot i - 1 \\ = -1 \text{ if } j = 2 \cdot i \\ = 0 \text{ otherwise} \end{cases}.$$

Lower memory bound



If $\mathcal{P}_1$ plays according to a Moore machine with less than 2^K states, he takes the same decision in some state t_x for the two highlighted prefixes (let $x = K$ w.l.o.g.).

$\Rightarrow \mathcal{P}_2$ can alternate and enforce decrease by 1 every two visits.

$\Rightarrow \mathcal{P}_1$ loses for any $v_0 \in \mathbb{N}^k$.

Symbolic synthesis algorithm

Algorithm CpreFP for MEPGs and MMPPGs:

- symbolic and incremental,
- winning strategy of at most exponential size,
- worst-case exponential time.

Symbolic synthesis algorithm

Algorithm CpreFP for MEPGs and MMPPGs:

- symbolic and incremental,
- winning strategy of at most exponential size,
- worst-case exponential time.

Idea: greatest fixed point of a $\text{Cpre}_{\mathbb{C}}$ operator.

- Exponential bound on the size of manipulated sets ($\sim$ width).
- Exponential bound on the number of iterations if a winning strategy exists ($\sim$ depth).

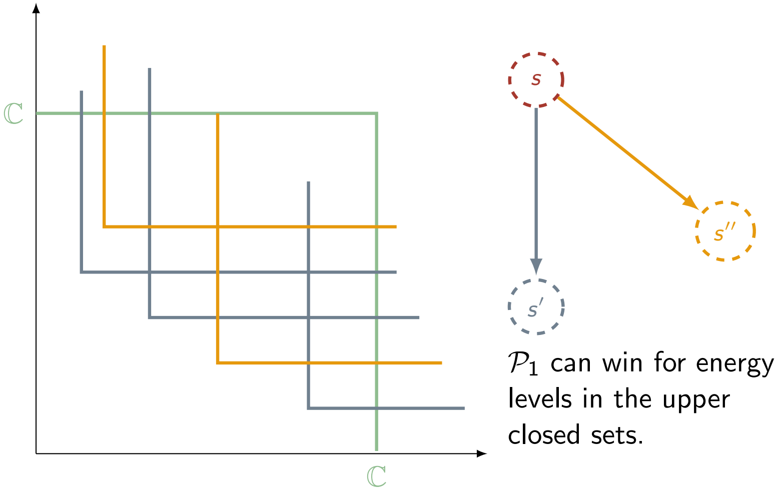
Symbolic synthesis algorithm: Cpre

- $\mathbb{C} = 2 \cdot l \cdot W \in \mathbb{N}$, $U(\mathbb{C}) = (S_1 \cup S_2) \times \{0, 1, \ldots, \mathbb{C}\}^k$,
- $\mathcal{U}(\mathbb{C}) = 2^{U(\mathbb{C})}$, the powerset of $U(\mathbb{C})$,
- $\mathsf{Cpre}_{\mathbb{C}} : \mathcal{U}(\mathbb{C}) \rightarrow \mathcal{U}(\mathbb{C})$, $\mathsf{Cpre}_{\mathbb{C}}(V) =$

$$\{(s_1, e_1) \in U(\mathbb{C}) \mid s_1 \in S_1 \wedge \exists (s_1, s) \in E, \exists (s, e_2) \in V : e_2 \leq e_1 + w(s_1, s)\}$$
$$\cup$$
$$\{(s_2, e_2) \in U(\mathbb{C}) \mid s_2 \in S_2 \wedge \forall (s_2, s) \in E, \exists (s, e_1) \in V : e_1 \leq e_2 + w(s_2, s)\}$$

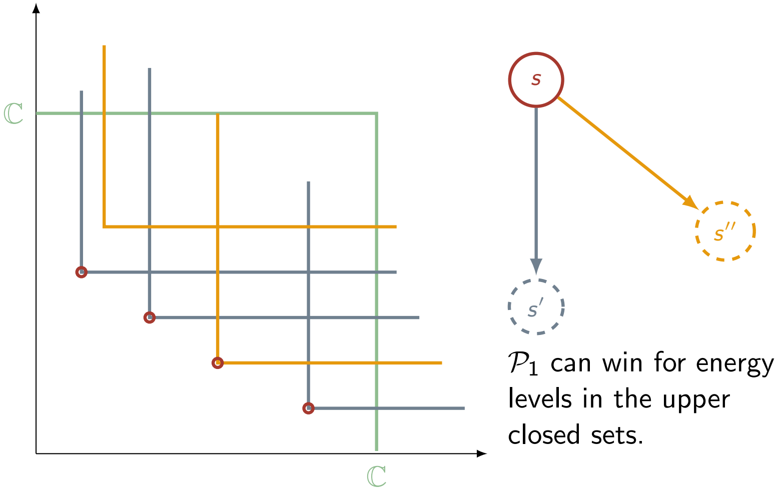
▷ Intuitively, compute for each state the set of winning initial credits, represented by the minimal elements of these upper closed sets.

Symbolic synthesis algorithm: Cpre



$\mathcal{P}_1$ can win for energy levels in the upper closed sets.

Symbolic synthesis algorithm: Cpre



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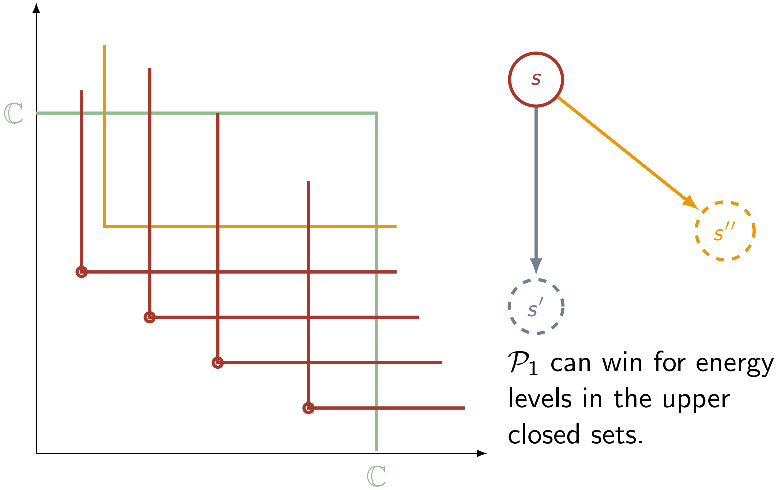

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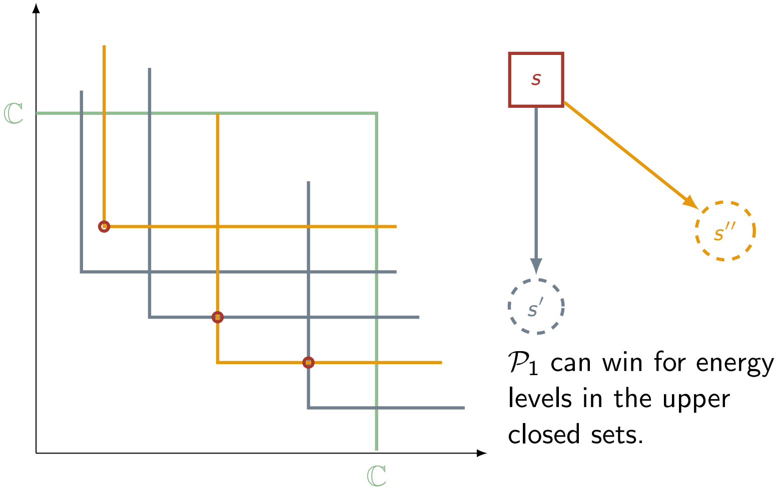

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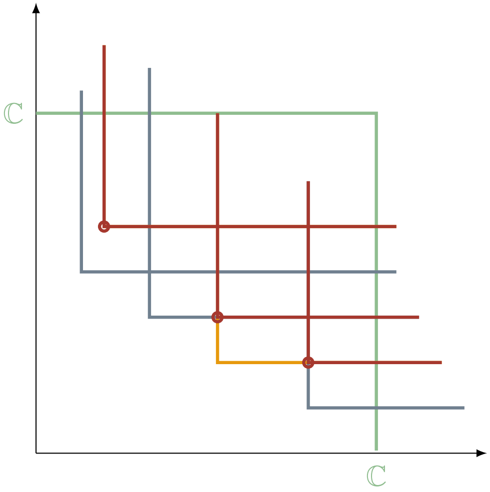
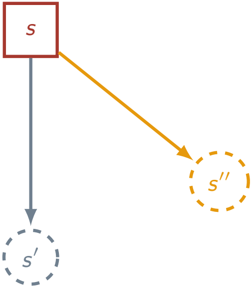




$\mathcal{P}_1$ can win for energy levels in the upper closed sets.

Symbolic synthesis algorithm: CpreFP

- **Correctness**
  - $(s_{init}, (c, \ldots, c)) \in \mathrm{Cpre}^*_{\mathbb{C}} \rightsquigarrow$ winning strategy for initial credit $(c, \ldots, c)$.
- **Completeness**
  - Winning strategy and SCT of depth $l \rightsquigarrow$ $(s_{init}, (\mathbb{C}, \ldots, \mathbb{C})) \in \mathrm{Cpre}^*_{\mathbb{C}}$ for $\mathbb{C} = 2 \cdot l \cdot W$.

Symbolic synthesis algorithm: CpreFP

- **Correctness**
  - ▷ $(s_{init}, (c, \ldots, c)) \in \mathsf{Cpre}_{\mathbb{C}}^{*} \rightsquigarrow$ winning strategy for initial credit $(c, \ldots, c)$.
- **Completeness**
  - ▷ Winning strategy and SCT of depth $l \rightsquigarrow$ $(s_{init}, (\mathbb{C}, \ldots, \mathbb{C})) \in \mathsf{Cpre}_{\mathbb{C}}^{*}$ for $\mathbb{C} = 2 \cdot l \cdot W$.

- Incremental approach over $\mathbb{C}$ can be used.
- Efficient implementation using antichains.

Obtained results

Question: *when and how can $\mathcal{P}_1$ trade his pure finite-memory strategy for an equally powerful randomized memoryless one ?*

	MEGs	EPGs	MMP(P)Gs	MPPGs
one-player	$\times$	$\times$	$\surd$	$\surd$
two-player	$\times$	$\times$	$\times$	$\surd$

- ▷ Sure semantics $\rightsquigarrow$ almost-sure semantics (i.e., probability one).

Obtained results

Question: *when and how can $\mathcal{P}_1$ trade his pure finite-memory strategy for an equally powerful randomized memoryless one ?*

	MEGs	**EPGs**	MMP(P)Gs	MPPGs
one-player	×	×	√	√
two-player	×	×	×	√

- ▷ Energy ∼ safety.
- ▷ Losing path ⇝ finite prefix witness ⇝ positive probability.
- ▷ Almost-sure winning ⇝ sure winning.

Obtained results

Question: *when and how can $\mathcal{P}_1$ trade his pure finite-memory strategy for an equally powerful randomized memoryless one ?*

	MEGs	EPGs	**MMP(P)Gs**	MPPGs
one-player	×	×	**√**	√
two-player	×	×	**×**	√

- ▷ One-player $\rightsquigarrow$ obtain the same edges frequencies through a probability distribution.
- ▷ Two-player $\rightsquigarrow$ no way to ensure balance against any strategy of $\mathcal{P}_2$ with an *a priori* fixed distribution.

Obtained results

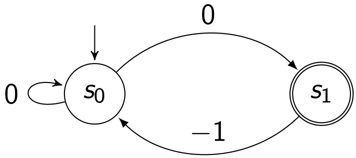
Question: *when and how can $\mathcal{P}_1$ trade his pure finite-memory strategy for an equally powerful randomized memoryless one ?*

	MEGs	EPGs	MMP(P)Gs	**MPPGs**
one-player	×	×	√	**√**
two-player	×	×	×	**√**

- ▷ First, show it for **mean-payoff Büchi games**.
- ▷ Then, induction on the number of priorities and the size of games, with subgames that reduce to the MP Büchi and MP coBüchi cases.

Mean-payoff Büchi games

Remark. MPBGs require infinite memory for optimality.



▷ $\mathcal{P}_1$ has to delay his visits of s_1 for longer and longer intervals.

Mean-payoff Büchi games

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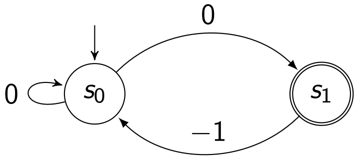
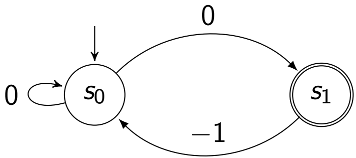


▷ $\mathcal{P}_1$ has to delay his visits of s_1 for longer and longer intervals.

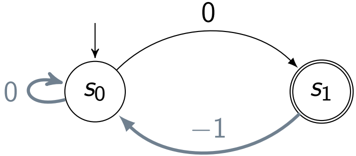
Lemma: *In MPBGs, ε-optimality can be achieved surely by pure finite-memory strategies and almost-surely by randomized memoryless strategies.*

MPBGs: sketch of proof



1. Let $G = (S_1, S_2, s_{init}, E, w, F)$, with F the set of Büchi states. Let $n = |S|$. Let Win be the set of winning states for the MPB objective with threshold 0 (w.l.o.g.). For all $s \in$ Win, $\mathcal{P}_1$ has two uniform memoryless strategies λ_1^{gfe} and $\lambda_1^{\Diamond F}$ s.t.

MPBGs: sketch of proof



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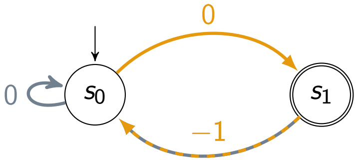
   - λ_1^{gfe} ensures that any cycle c of its outcome have $\mathrm{EL}(c) \geq 0$ [CD10],

MPBGs: sketch of proof



1. Let $G = (S_1, S_2, s_{init}, E, w, F)$, with F the set of Büchi states. Let $n = |S|$. Let Win be the set of winning states for the MPB objective with threshold 0 (w.l.o.g.). For all $s \in$ Win, $\mathcal{P}_1$ has two uniform memoryless strategies λ_1^{gfe} and $\lambda_1^{\Diamond F}$ s.t.
   - λ_1^{gfe} ensures that any cycle c of its outcome have $\text{EL}(c) \geq 0$ [CD10],
   - $\lambda_1^{\Diamond F}$ ensures reaching F in at most n steps, while staying in Win.

MPBGs: sketch of proof

2 For $\varepsilon > 0$, we build a pure finite-memory λ_1^{pf} s.t.

(a) it plays λ_1^{gfe} for $\frac{2 \cdot W \cdot n}{\varepsilon} - n$ steps, then

(b) it plays $\lambda_1^{\Diamond F}$ for n steps, then again (a).

MPBGs: sketch of proof

2. For $\varepsilon > 0$, we build a pure finite-memory λ_1^{pf} s.t.
   (a) it plays λ_1^{gfe} for $\frac{2 \cdot W \cdot n}{\varepsilon} - n$ steps, then
   (b) it plays $\lambda_1^{\Diamond F}$ for n steps, then again (a).

   This ensures that

   - F is visited infinitely often,
   - the total cost of phases (a) + (b) is bounded by $-2 \cdot W \cdot n$, and thus the mean-payoff is at least $-\varepsilon$.

MPBGs: sketch of proof

3. Based on λ_1^{gfe} and $\lambda_1^{\Diamond F}$, we obtain almost-surely ε-optimal *randomized memoryless* strategies, i.e.,

$$\forall \varepsilon > 0,\ \exists \lambda_1^{rm} \in \Lambda_1^{RM},\ \forall \lambda_2 \in \Lambda_2,$$
$$\mathbb{P}_{s_{init}}^{\lambda_1^{rm},\lambda_2}(\mathrm{Par}(\pi) \bmod 2 = 0) = 1 \ \wedge\ \mathbb{P}_{s_{init}}^{\lambda_1^{rm},\lambda_2}(\mathrm{MP}(\pi) \geq -\varepsilon) = 1.$$

MPBGs: sketch of proof

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Strategy:

$$\forall s \in S,\ \lambda_1^{rm}(s) = \begin{cases} \lambda_1^{gfe}(s) \text{ with probability } 1-\gamma, \\ \lambda_1^{\Diamond F}(s) \text{ with probability } \gamma, \end{cases}$$

for some *well-chosen* $\gamma \in]0,1[$.

MPBGs: sketch of proof

Büchi

- Probability of playing as $\lambda_1^{\Diamond F}$ for n steps in a row and ensuring visit of F strictly positive at all times.
- Thus λ_1^{rm} almost-sure winning for the Büchi objective.

MPBGs: sketch of proof

Mean-payoff

- Consider
  - all end components
  - in all MCs induced by pure memoryless strategies of $\mathcal{P}_2$.
- Choose γ so that all ECs have expectation $> -\varepsilon$.
- Put more probability on lengthy sequences of *gfe* edges.

Conclusion

- Quantitative objectives
- Parity
- Restriction to finite memory (practical interest)
- Exponential memory bounds
- EXPTIME symbolic and incremental synthesis
- Randomness instead of memory

Results Overview

- Memory bounds

MEPGs	MMPPGs	
optimal	finite-memory optimal	optimal
exp.	**exp.**	infinite [CDHR10]

- Strategy synthesis (finite memory)

MEPGs	MMPPGs
EXPTIME	**EXPTIME**

- Randomness as a substitute for finite memory

	MEGs	EPGs	MMP(P)Gs	MPPGs
one-player	×	×	√	√
two-player	×	×	×	√

Ongoing work

- Consider alternative, more natural definition of MP-like objective, with possibly good synthesis properties.

Thanks. Questions ?

Roderick Bloem, Krishnendu Chatterjee, Thomas A. Henzinger, and Barbara Jobstmann.
Better quality in synthesis through quantitative objectives.
In *Proc. of CAV*, volume 5643 of *LNCS*, pages 140–156. Springer, 2009.

Patricia Bouyer, Ulrich Fahrenberg, Kim Guldstrand Larsen, Nicolas Markey, and Jirí Srba.
Infinite runs in weighted timed automata with energy constraints.
In *Proc. of FORMATS*, volume 5215 of *LNCS*, pages 33–47. Springer, 2008.

Tomás Brázdil, Petr Jancar, and Antonín Kucera.
Reachability games on extended vector addition systems with states.
In *Proc. of ICALP*, volume 6199 of *LNCS*, pages 478–489. Springer, 2010.

Patricia Bouyer, Nicolas Markey, Jörg Olschewski, and Michael Ummels.
Measuring permissiveness in parity games: Mean-payoff parity games revisited.
In *Proc. of ATVA*, volume 6996 of *LNCS*, pages 135–149. Springer, 2011.

Krishnendu Chatterjee and Laurent Doyen.
Energy parity games.
In *Proc. of ICALP*, volume 6199 of *LNCS*, pages 599–610. Springer, 2010.

Arindam Chakrabarti, Luca de Alfaro, Thomas A. Henzinger, and Mariëlle Stoelinga.
Resource interfaces.
In *Proc. of EMSOFT*, volume 2855 of *LNCS*, pages 117–133. Springer, 2003.

Krishnendu Chatterjee, Laurent Doyen, Thomas A. Henzinger, and Jean-François Raskin.
Generalized mean-payoff and energy games.
In *Proc. of FSTTCS*, volume 8 of *LIPIcs*, pages 505–516. Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2010.

Krishnendu Chatterjee, Thomas A. Henzinger, and Marcin Jurdzinski.
Mean-payoff parity games.
In *Proc. of LICS*, pages 178–187. IEEE Computer Society, 2005.

A. Ehrenfeucht and J. Mycielski.
Positional strategies for mean payoff games.
International Journal of Game Theory, 8(2):109–113, 1979.

Uli Fahrenberg, Line Juhl, Kim G. Larsen, and Jirí Srba.
Energy games in multiweighted automata.

In *Proc. of ICTAC*, volume 6916 of *LNCS*, pages 95–115. Springer, 2011.

T.M. Liggett and S.A. Lippman.
Short notes: Stochastic games with perfect information and time average payoff.
Siam Review, 11(4):604–607, 1969.

Charles Rackoff.
The covering and boundedness problems for vector addition systems.
Theor. Comput. Sci., 6:223–231, 1978.

Louis E. Rosier and Hsu-Chun Yen.
A multiparameter analysis of the boundedness problem for vector addition systems.
J. Comput. Syst. Sci., 32(1):105–135, 1986.