Extending finite-memory determinacy by Boolean combination of winning conditions

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June 22, 2019

MoRe 2019 – 2nd International Workshop on Multi-objective Reasoning in Verification and Synthesis

The talk in one slide

Strategy synthesis for two-player games on graphs

Finding **good** controllers for systems interacting with an *antagonistic* environment.

- Good? Performance evaluated through *objectives* / *payoffs*.

Question

When are *simple* strategies sufficient to play optimally?

- We establish a general framework that preserves **finite-memory determinacy** when combining objectives.
- Joint work with S. Le Roux and A. Pauly, in FSTTCS'18 [RPR18] (on arXiv).

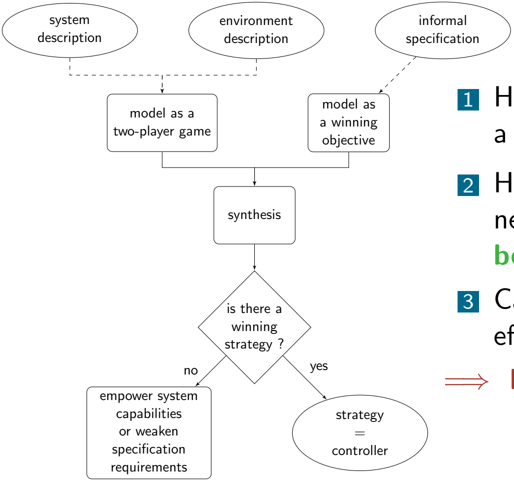
1 Context, games, strategies

2 Memoryless determinacy

3 Finite-memory determinacy and Boolean combinations

4 Conclusion and ongoing work

Strategy synthesis for two-player games



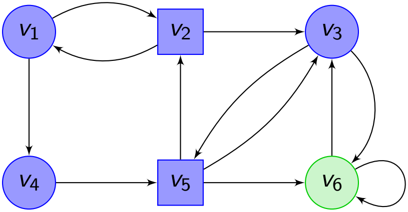
1. How complex is it to decide if a winning strategy exists?
2. How complex such a strategy needs to be? **Simpler is better**.
3. Can we synthesize one efficiently?

$\Longrightarrow$ **Focus on Question 2**.

Games on graphs: example

We consider *finite* arenas with vertex *colors* in C. Two players: circle (1) and square (2). Strategies $C^* \times V_i \to V$ (w.l.o.g.).

- ▷ A **winning condition** is a set $W \subseteq C^\omega$.



From where can Player 1 ensure to reach v_6? How complex is his strategy?

Memoryless strategies ($V_i \to V$) always suffice for reachability (for both players).

When are memoryless strategies sufficient to play optimally?

Virtually always for **simple** winning conditions!

Examples: reachability, safety, Büchi, parity, mean-payoff, energy, total-payoff, average-energy, etc.

Can we characterize when they are?

Yes, thanks to Gimbert and Zielonka [GZ05] (see also, e.g., [Kop06, AR17]).

Gimbert and Zielonka's criterion

Memoryless strategies suffice for a *preference relation* (and the induced winning conditions) iff

1. it is **monotone**,
   - ▷ Intuitively, stable under prefix addition.
2. it is **selective**.
   - ▷ Intuitively (the true characterization is slightly more subtle), stable under cycle mixing.

Example: reachability.

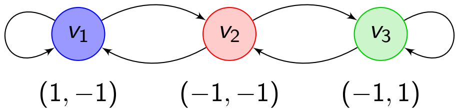
No equivalent for finite memory!

I will come back to that. . . ☺

Combining winning conditions (1/2)

Multi-objective reasoning is crucial to model trade-offs and interplay between several qualitative and quantitative aspects.

Memoryless strategies do not suffice anymore, even for simple conjunctions!



Examples:

- Büchi for v_1 *and* v_3 → **finite** (1 bit) memory.
- Mean-payoff (average weight per transition) ≥ 0 on all dimensions → **infinite** memory!

Combining winning conditions (2/2)

Our goal

We want a *general* and *abstract* theorem guaranteeing the sufficiency of finite-memory strategies[a] in games with Boolean combinations of objectives provided that the underlying simple objectives fulfil some criteria.

[a] Implementable via a finite-state machine.

Advantages:

- study of core features ensuring finite-memory determinacy,
- works for almost all existing settings and many more to come.

Drawbacks:

- concrete memory bounds are huge (as they depend on the most general upper bound).
- sufficient criterion, not full characterization.

The building blocks

The full approach is technically involved but can be sketched intuitively.

Criterion outline

Any *well-behaved* winning condition combined with conditions traceable by finite-state machines (i.e., *safety-like* conditions) preserves finite-memory determinacy.

To state this theorem formally, we need three ingredients:

1. *regularly-predictable* winning conditions,
2. *regular* languages,
3. *hypothetical* subgame-perfect equilibria (hSPE).

Regular predictability

Regularly-predictable winning condition

A winning condition is regularly-predictable if for all games, for all vertices, there exists a finite automaton that recognizes the color histories from which Player 1 has a winning strategy.

- All prefix-independent objectives are regularly-predictable.
- Reachability and safety are not prefix-independent but are regularly-predictable.

Regular-predictability $\neq$ FM determinacy!

- ▷ Energy games with only a lower bound are memoryless determined but not regularly-predictable.
- ▷ Let W be the non-regular sequences in $\{0, 1\}^\omega$: it is prefix-independent hence regularly-predictable but finite-memory strategies do not suffice to win.

Regular combinations of winning objectives

Let $\mathcal{W}$ be a class of winning conditions closed under Boolean combinations (can be the trivial one).

We denote by $R_\ell(\mathcal{W})$ the set of winning objectives obtained by Boolean combination of objectives in $\mathcal{W}$ and ℓ safety-like conditions based on regular languages over C (i.e., conditions asking that there is no prefix of the play in the regular language).

Examples: fully-bounded energy conditions and window conditions can be described as regular languages, hence added freely in Boolean combinations with more general objectives.

Remark

Regular conditions are regularly-predictable, not the opposite.

Hypothetical subgame-perfect equilibria

A *strategy profile* where both players play optimally after all initial histories

- that are possible from the starting vertex in the arena is called a *subgame-perfect equilibrium (SPE)*.
- in C^* is called a **hypothetical** SPE.

HSPEs are technically useful when combining games.

FM hSPE slightly more restrictive than FM determinacy.

Morally equivalent in almost all settings.

$\Longrightarrow$ **We will see a corner case later.**

Our main result (sketch)

Regular combinations preserve FM determinacy

Let $\mathcal{W}$ be a class of winning conditions that

1. is closed under Boolean combinations,
2. is regularly-predictable,
3. ensures the existence of finite-memory hSPE.

Then all conditions in $R_\ell(\mathcal{W})$ also satisfy properties 2 and 3.

If you think of it as combinations with safety-like conditions, not surprising. . .

But finding the good concepts and proving the result was difficult!

Rediscovery of FM determinacy results (1/2)

Regular conditions: reachability, safety, fully-bounded energy, window (mean-payoff and parity), etc.

Regularly-predictable conditions.

- *Regular ones.* Multi-dimension fully-bounded energy games [BFL+08, BMR+18, BHM+17], conjunctions of window objectives [CDRR15, BHR16a], extension to Boolean combinations.
- *Parity and Muller.* Combinations expressible in the closed class, can be mixed in any Boolean combination with regular languages and retain FM determinacy.
  Generalized parity games [CHP07], or combinations of parity conditions with window conditions [BHR16b], extension to Boolean combinations.

Rediscovery of FM determinacy results (2/2)

- *Mean-payoff.* Regularly-predictable and admits FM hSPE. Not true for Boolean combinations [VCD+15, Vel15]. One can take $\mathcal{W}$ as the trivial class containing one mean-payoff condition and its complement, and use it in Boolean combinations with regular languages.
- *Average-energy, total-payoff and energy with no upper bound.* Not regularly-predictable as one needs to be able to store an arbitrarily large sum of weights in memory to decide if Player 1 can win from a given prefix. Hence our theorem cannot be applied to these conditions.

Theorem applicability

Some conditions we do not cover

Combinations of mean-payoff, average-energy, total-payoff, or combinations of mean-payoff and parity.

But they do not preserve FM determinacy!
[VCD+15, Vel15, BMR+18, CDRR15, CHJ05]

And we rediscover many results from the literature [BFL+08, BMR+18, BHM+17, CDRR15, BHR16a, CHP07, BHR16b] **and are able to extend them to more general combinations (or to completely novel ones).**

Corner cases: FM determined combinations we do not cover

We know of three cases:

1. conjunctions of energy conditions [CRR14, JLS15],
2. conjunctions of energy and parity conditions [CD12, CRR14],
3. conjunctions of energy and a single average-energy condition [BHM+17].

Observation: common technique in ad-hoc proofs

Proving equivalence with games where the energy condition can be bounded *both* from below *and from above*, for a sufficiently large bound.

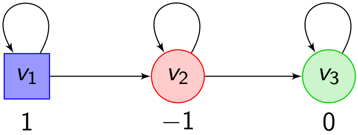
$\Longrightarrow$ **We retrieve applicability of our theorem for cases 1 and 2.**

Focus: average-energy + energy conditions

Only case of preservation of FM determinacy which we do not cover!

- The average-energy condition is not regularly-predictable [BMR+18, BHM+17].
- And it behaves rather oddly in comparison to all other classical objectives. . . .

Average-energy games with a lower-bounded energy condition are FM determined but do not admit FM hSPE, the only setting in this case to our knowledge.



Goal: reach v_3 with sum zero.

- FM determined.
- SPE require infinite memory.

Conclusion

- Combining similar simple objectives leads to contrasting behaviors: difficult to extract the core features leading to FM determinacy.
- Our main result is a **sufficient criterion**, not a full characterization.
  - ▷ In practice, it does cover everything except *average-energy with a lower-bounded energy condition* – a very strange corner case.
  - ▷ **Any weakening of our hypotheses almost immediately leads to falsification.**
  - ▷ We also have several **more precise results** (e.g., much lower bounds) for specific combinations and/or restrictive hypotheses.

Ongoing work

We now have an almost complete picture of the frontiers of FM determinacy for *combinations of objectives*.

What about a complete characterization à la Gimbert and Zielonka?

Ongoing work with P. Bouyer, S. Le Roux, Y. Oualhadj and P. Vandenhove. Promising preliminary results.

Thank you! Any question?

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